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ON-LINE LEAK MONITORING IN FLUID PUMPING SYSTEMS

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Abstract

In this paper, a model-based leak detection methodology for fluid pumping systems is developed. The novelty of this approach lies in modelling the leak position as a point between a real and an imaginary valve. The equations that describe the resulting dynamic system are then put into an input-output form suitable for least squares estimation. In this way, classic parameter-estimation based detection methods are applied to moving windows of system data. Computer simulation illustrates the feasibility of the method.

Key Words

Leak detection, fault monitoring, reliability

1. Introduction

The supervision of technical processes is the focus of increased development because of today's high demands on reliability and safety. Latest trends toward high autonomy systems, as advocated by Antsaklis and co-workers [1] and Zeigler et al. [2] among others, make the introduction of sophisticated fault detection methods an essential stage in the design of modern control systems. One of the areas in which these ideas have been applied is fluid dynamic systems. These systems, which either handle or employ fluids as working media, play a very important role in modern industry. They are encountered in process industries, power generation, aeronautical applications, air conditioning or heating of buildings, and so on. They are implemented in simple forms of pumping circuits, in more complex piping networks and even gas turbines.

Leak monitoring in fluid systems is important because of safety, environmental, and economic issues. The subject of the present paper is the development of a leak monitoring method for certain types of such systems, namely systems containing one turbomachinery component, pump, or compressor. The importance of this work stems from the fact that although techniques for the performance study of such systems have been developed, they have been used for condition monitoring in a very fundamental manner. On the other hand, although diagnostic techniques have tremendously progressed in recent years, they have mainly dealt with electrical and electronic systems, while those applied to mechanical systems have developed along rather independent paths.

Leak detection methods may be classified according to their time characteristics, into *intermittent off-line testing* and *continuous on-line supervision.* Some typical off-line methods include *hydrostatic pressure testing, sonic pig testing,* and *direct inspection.* These simple techniques can detect tiny long-term leaks, but at the expense of detection speed.

On-line detection methods attempt to overcome these shortcomings. They can also be subdivided into *conventional* and *advanced* leak detection methods. The former rely on static process models and are mainly based on balancing the input and output flows. However, leaks smaller than 2% of the total flow for liquids and 10% for gases cannot be detected by these methods due to noise effects and inherent dynamics. An added difficulty is the unavailability of adequate instrumentation, which is limited to flow-rate and pressure sensors. Furthermore, transient effects are not taken into account. Advanced leak detection methods are based on comprehensive dynamic process models using available on-line measurements.

Previous attempts in this direction are presented in the surveys of Iserman [3] and Lappus and Schmidt [4]. Iserman and Siebert [5], proposed a non-parametric cross-correlation in which leak detection is performed by cross-correlating the differences:

$$\Delta\dot{M}_0(k) = \dot{M}_0(k) - \dot{M}_0^*(k)$$
$$\Delta\dot{M}_1(k) = \dot{M}_1(k) - \dot{M}_1^*(k)$$

where $\dot{M}_0(k)$, $\dot{M}_1(k)$ denote the flow rates at the inlet and exit, and the starred quantities are obtained by discrete-time low pass filtering. Leak location and flow is then estimated by calculation of the point of intersection of the pressure curves. Experimental results in a 65km pipeline demonstrated the ability of this method to detect liquid leaks of about 0.19% after 98s of their occurrence and estimate their location within ±500m (0.7%) after 188s. Schmidt et al. [6] proposed a leak detection scheme for gas transport pipelines based on a Luenberger observer. Lappus [7] extended this approach to gas transmission pipelines. Among the first to include an explicit model of the fault, Digernes describes a multiple model hypothesis testing method, employing parallel Kalman filters, for leak detection in an oil pipeline 30km long [8]. In this approach each filter models a leak at a predefined location. A leak of 1% of the total flow is reported to be detected after 160s. Benkherouf et al. [9] used a single extended Kalman filter in a model which included artificial leak states at prespecified points.

In this paper, a parameter estimation based method for the detection of leaks in fluid pumping systems is

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proposed. A dynamic model of the fluid system is utilized, with explicit modelling of the leak size and location. The estimation algorithm is a recursive sliding window least squares method, with low computational load and fast detection rates, making it very suitable for low-cost on-line applications.

2. Fluid Pumping System Dynamic Modelling

An essential element of the proposed method of leak monitoring is the development of an accurate mathematical model of the dynamics of the supervised fluid system. In order to build such a model, the system is subdivided into components and the function of each component is described through the appropriate equations. The overall model is then derived through application of the conditions imposed by the requirement of matching the components in the particular circuit layout.

In this investigation, we will consider systems that are using an incompressible fluid and contain one turbocomponent i.e., a turbopump. The elements that constitute such systems are: pipes, tanks and obstruction elements such as valves, bends, contractions, etc. In particular, let the pumping system of fig. 1 be considered.

Figure 1

The novel idea in this configuration is the fact that the unknown leak point is positioned between an actual and an imaginary valve. It is shown in Appendix 1 that the dynamic equation describing the behaviour of this system is:

$$\frac{\partial M_1}{\partial t} = c_1 M_1^2(t) + c_2 f(M_1, \omega) \tag{1a}$$

$$\frac{M_1(t) - M_6(t)}{M_1(t)} = \sigma (1b)$$

where:

$$c_1 = \frac{\xi_1 + \frac{\lambda l_{1L}}{D} + \left(1 + \xi_2 + \frac{\lambda l_{L6}}{D}\right)(1 - \sigma^2)}{2\rho\left(\sigma l_{1L} + l(1-\sigma)\right) A} \tag{2}$$

$$c_2 = \frac{A}{\sigma l_{1L} + l(1-\sigma)} \tag{3}$$

and the physical parameters denote:

$M_i(t)$: fluid flow rate at point i (kg s^{-1})
$\omega(t)$: pump rotational speed (s^{-1})
$f(M_1, \omega)$: characteristic of pump
ρ: fluid density (kg/m^3)
λ: friction coefficient
σ: leak flow rate as percentage of M_1 (%)
D: diameter of pipe (m)
A: cross-sectional area of pipe (m^2)
ξ_1: loss coefficient of valve positioned before the leak
ξ_2: loss coefficient of valve positioned after the leak
l: total length of pipe (m)
l_{1L}: length of pipe from point 1 to leak point (m)
l_{L6}: length of pipe from leak point to point 6 (m)

In particular the characteristic of the pump is expressed in the form:

$$f\left(M_1(t), \omega(t)\right) = h_1 M_1^2(t) + h_2 M_1(t)\omega(t) + h_3\omega^2(t) \tag{4}$$

where the h_i are appropriate known pump constants. This expression covers the whole range of characteristics of the pump. In this way $\omega(t)$ is taken as the input to the system and (1a) and (1b) can be conveniently written as:

$$\frac{\partial M_1}{\partial t} = \left[M_1^2(t)\,\omega(t)M_1(t)\,\omega^2(t)\right]\begin{bmatrix}\theta_{11}\\ \theta_{12}\\ \theta_{13}\end{bmatrix} \tag{5a}$$

$$\frac{M_1(t) - M_6(t)}{M_1(t)} = \theta_{21} \tag{5b}$$

where:

$$\theta_{11} = \frac{\xi_1 + \frac{\lambda l_{1L}}{D} + \left(1 + \xi_2 + \frac{\lambda l_{L6}}{D}\right)(1 - \sigma^2)}{2\rho\left(\sigma l_{1L} + l(1-\sigma)\right) A} + \frac{A h_1}{\sigma l_{1L} + l(1-\sigma)} \tag{6a}$$

$$\theta_{12} = \frac{A h_2}{\sigma l_{1L} + l(1-\sigma)} \tag{6b}$$

$$\theta_{13} = \frac{A h_3}{\sigma l_{1L} + l(1-\sigma)} \tag{6c}$$

$$\theta_{21} = \sigma \tag{6d}$$

Equations (5a) and (5b) are linear-in-the-parameters differential and algebraic equations, respectively, and therefore, the parameter vectors θ_i can be estimated from input-output data using the appropriate techniques.

3. Leak Monitoring

A well-designed leak monitoring system should perform the following tasks [4]:

- detection of leak
- estimation of the leak location
- estimation of the leak flow rate
- reorganization following a positive leak decision (closing valves, etc.)

The performance of a leak monitoring system can be ascertained from the following requirements:

- the size of detectable leak should be as small as possible
- the time to detection should be as short as possible
- the required duration of the leak should be small
- the probability of false alarms should be negligible (<0.005)

Since leak size is important, leaks are classified as:

- pinhole leaks: $\sigma < 1\%$ of the average pipeline flow
- small-size leaks: $1\% < \sigma < 10\%$
- large-size leaks: $\sigma > 10\%$

In the proposed method, attention is given to all the above requirements. The different tasks will now be specified.

3.1 Leak Detection

As mentioned earlier, the dynamics of the process are put in the form of a linear-in-the-parameters differential equation. If measurements of M_1 and ω are taken at discrete time instants, then for every measurement pair, $y_1(k), y_2(k)$, (5a) and (5b) can be written in discrete time as follows:

$$y_i(k) = \mathbf{u}_i^T(k)\theta_i + e_i(k); k = 0, t_h, \ldots, nt_h, \ldots \quad (7)$$

where:

$$y_1(k) \equiv \frac{\partial M_1}{\partial t}\Big|_{t=k} \qquad \text{(output data)}$$

$$y_2(k) \equiv \frac{M_1(t) - M_6(t)}{M_1(t)} \qquad \text{(output data)}$$

$$\mathbf{u}_1(k) \equiv \lfloor M_1^2(k)\,\omega(k) M_1(k)\,\omega^2(k)\rfloor \qquad \text{(input data)}$$

$$u_2(k) \equiv 1 \qquad \text{(input data)}$$

$$\theta_1 = [\theta_{11}\,\theta_{12}\,\theta_{13}] \qquad \text{(unknown parameter vector)}$$

$$\theta_2 = [\theta_{21}] \qquad \text{(unknown parameter vector)}$$

Also t_h is the sampling interval and $e_i(k)$ a random term representing modelling and measurement errors.

The parameter estimation method used is a recursive, moving window, least squares method [10]. This method provides fast detection times due to the fact that only the most recent window of n_w data pairs is used for the parameter estimation. Furthermore, its recursive nature makes it especially suitable for on-line applications. The relevant equations applied twice for $\mathbf{theta}_1, \theta_2$ are:

$$\hat{\theta}(k+1) = \hat{\theta}(k) - \mathbf{P}(k+1)\left[\mathbf{\Gamma}(k+1)\hat{\theta}(k) - \delta(k+1)\right] \quad (8a)$$

$$\mathbf{P}^{-1}(k+1) = \mathbf{P}^{-1}(k) + \mathbf{\Gamma}(k+1) \quad (8b)$$

where:

$$\mathbf{\Gamma}(k+1) = \mathbf{u}(k+1)\mathbf{u}^T(k+1) - \mathbf{u}(k-n_w+1)\mathbf{u}^T(k-n_w+1) \quad (8c)$$

$$\delta(k+1) = \mathbf{u}(k+1)\mathbf{y}(k+1) - \mathbf{u}(k-n_w+1)\mathbf{y}(k-n_2+1) \quad (8d)$$

Here n_w is the moving window length, $\mathbf{u}(k+1)$, $y(k+1)$ are the current measurements entering the moving window, and $\mathbf{u}(k-n_w+1)$, $y(k-n_w+1)$ are the discarded measurements leaving the moving window. The equations are initialized by their normal (no-fault) operation values.

Once the vector θ_1 and scalar θ_2 have been estimated, the physical parameters of interest must be calculated. Unfortunately (6b) and (6c) alone are not sufficient to calculate $\hat{l}_{1L}$ and $\hat{\sigma}$ simultaneously, since they both reduce to the form:

$$\hat{\sigma}(l - \hat{l}_{1L}) = \hat{\sigma}\hat{l}_{L6} = l - \frac{Ah_i}{\hat{\theta}_{li}}; \; i = 2, 3 \quad (9)$$

Furthermore (6a) requires knowledge of the ξ_i's whose values (0 or ξ) depend on the leak position and hence cannot be considered known. Therefore, measurement of $M_6(t)$ must be made, which when inserted in (5b) gives $\hat{\sigma}$. Subsequently, $\hat{l}_{1L}$ can be calculated from either (6b) or (6c). Equation (6a) can be used to verify the results.

Some implementation issues must also be made. Firstly, the derivative $\partial M_1/\partial t$ required in the procedure, is usually calculated either numerically or by the use of *state variable filters* [11]. The former approach is adopted in this study and the 9-point central difference formula is used, given by:

$$y'(t) = (1/840h)[3y(t-4h) - 32y(t-3h)$$
$$+168y(t-2h) - 672y(t-h) + 672y(t+h)$$
$$-168y(t+2h) + 32y(t+3h) - 3y(t+4h)]$$

where h is the step length.

Secondly, the window length, n_w, must be specified. This is usually selected by simulation, since its value affects the quality of the estimates and consequently the probability of correct detections and false alarms. On the other hand, too large a value will result in increased detection delay time (decreased sensitivity), since a leak will take a longer time to affect the estimates. In such cases, a compromise is made based on the situation at hand. Note, however, that because of the recursive nature of the algorithm given by (8), window size does not affect the calculation speed. Thirdly, it is assumed that all other parameters of (5a) remain constant, i.e., no fault that affects their values takes place. Situations where this is not the case are the subject of ongoing research.

Lastly, the form of (7) is not a realistic one. In practice, noise will enter into the measurements of $M_1(t)$ and $\omega(t)$ modifying the observation equation. In that case, instead of (7), we will have:

$$y(k) = \Big[(M_1(k) + v_M(k))^2\,(\omega(k) + v_\omega(k))$$
$$(M_1(k) + v_M(k))\,(\omega(k) + v_\omega(k))^2\Big]^T \theta_1$$
$$= \lfloor M_1^2(k) + v_M^2(k) + 2M_1(k)v_M(k)\omega(k)M_1(k)$$
$$+v_\omega(k)M_1(k) + \omega(k)v_M(k) + v_\omega(k)v_M(k)$$

$$\omega^2(k) + v_\omega^2(k) + 2\omega(k)v_\omega(k)\rfloor\theta_1 \tag{10a}$$

$$\sigma = \frac{(M_1(k)+v_M(k)) - (M_6(k)+w_M(k))}{(M_1(k)+v_M(k))} \tag{10b}$$

where $v_M(k)$, $w_M(k)$, $v_\omega(k)$ are Gaussian measurement noise for the pressures and angular velocity respectively, having zero means and standard deviations s_M, and s_ω. Furthermore, since $y(k)$ is produced by numerically differentiating $M_1(t)$, the noise will enter into this operation as well. Neglecting any numerical errors, we will then, in effect, compute:

$$y(k) = \frac{\partial}{\partial t}\left(M_1(k) + v_m(k)\right) \tag{11}$$

The situation of noisy regressors is well known, and can be treated in the case of ARX systems, for example, by the use of Instrumental Variable methods [12]. However, this case is much more complicated and Instrumental Variable methods are not applicable. The proposed procedure relies on the following:

If a pre-filter window of n_f pairs of input-output data are gathered and the mean value of (10a) over these measurements is taken, it is shown in Appendix 2 that θ_1 and σ can be calculated from the equations:

$$\frac{1}{n_f}\sum_{i=k}^{k_f}\frac{\partial}{\partial t}M_1(i) =$$

$$\left(\frac{1}{n_f}\sum_{i=k}^{k_f}\left[M_1^2(i)\omega(i)M_1(i)\omega^2(i)\right] - c_n\left[\hat{s}_M^2\;0\;\hat{s}_\omega^2\right]\right)\theta_1$$

$$1-\sigma = \frac{\frac{1}{n_f}\sum_{i=k}^{k+n_f-1} M_6(i)}{\frac{1}{n_f}\sum_{i=k}^{k+n_f-1} M_1(i)}$$

where $c_n = n_f/(n_f-1)$, and $k_f = k + n_f - 1$.

Once the estimates $\hat{l}_{1L}$, $\hat{\sigma}$ of l_{1L} and σ have been obtained, a statistical procedure must be employed to decide whether a leak has occurred or not. Such a procedure is described by Stavrakakis et al. [13]. In the present situation however, a leak can be declared if $\hat{\sigma}$ exceeds a predefined limit. This limit is obtained by considering both the accuracy of the flow measuring device and the desired threshold of leak flow.

3.2 Leak Size and Position Estimation

Following a leak detection, the leak size and position estimation phase is entered. In this phase, a normal (non-window) least squares estimation procedure is used, which is started at the detection instant. The initial values used are the system's estimates at the detection instant for θ_i, $i = 1, 2$ and a large diagonal matrix for the estimates' covariance matrix reflecting the high initial uncertainty. In this way, it is assured that only data that contain the faulty parameters are used, so that the estimate's accuracy is improved. The amount of data employed in this phase depends on the required accuracy, but 2000 points should be adequate for most applications.

4. Simulation Study

To demonstrate the effectiveness of the proposed method, the system of fig. 1 with the following parameter values was simulated on a microcomputer: $l = 130$m, $D = 0.07$m, $\rho = 1000$kg/m^3, $\xi = 2.25$, and $\lambda = 0.0035$. A valve was placed at a distance of $l_{12} = 5$m from the beginning of the pipe, while the pump was operated using (4) with h_1 = -314.97, $h_2 = 0.316$, and $h_3 = 0.02053$. An artificial leak of $\sigma = 2\%$ developed at $t = 500$sec at a point $l_{1l} =$ 60m resulting in $\xi_1 = 2.25$ and $\xi_2 = 0$. As input to the system, a harmonic signal given by $\omega(t) = 1500 + 500\sin(\phi t/25)$ was applied. The system was simulated for 2000 sec. Equation (5a) was solved using a fifth-order Runge-Kutta with variable step length [14]. For the numerical differentiation, the nine-point formula with $h = 0.5$ was used. Different combinations of noise variables and pre-filter sizes were examined and results from some of the runs are shown in figs. 3–7. In all cases the detection window size was $n_w = 100$.

Figures 3–7

In figs. 3–4, the results of a typical simulation with no measurement errors are shown. In this case noise was applied directly to the model equations. The noise variances were var$(e_1(k)) = (0.005)^2$ and var$(e_2(k)) = (0.05)^2$. In figs. 5–6, the results for the same system but with measurement errors are shown. Here var$(v_M(k)) = (0.001)^2$, var$(v_\omega(k)) = (0.01)^2$, and $n_f = 30$. As seen, the proposed system works satisfactorily in both cases, giving excellent leak size and position estimates. In the case of a real system, greater accuracy is expected since the sample will exceed the 2000 mark used in the simulations. Furthermore, it must be emphasized that the harmonic test input used is the worst possible case and covers every combination of actual inputs. In these cases, better examples are also expected.

As seen from the simulations, estimates of leak size σ are much better than estimates of leak position. This is because leak position is estimated indirectly using (9) for $I = 3$. Its accuracy depends both on the quality of the estimate of θ_3 and the system parameters D and h_3 since:

$$\delta\hat{l}_{L6} = \delta\hat{\theta}_{13}\frac{Ah_3}{\hat{\theta}_{13}^2}$$

It may be argued that the noise variances used are small. However, the RHS of (5a) is also small as shown in fig. 2 and there are instants when the noise term completely covers the output signal $y(t)$.

5. Conclusions

A model-based leak detection system for fluid pumping systems is developed. With the proposed method, both the leak size and leak position are estimated by a simple and fast estimation procedure suitable for on-line implementation. The problem of direct measurement noise in the regressors, inherent in all practical applications, is also addressed and a viable solution is proposed. The whole system is successfully tested by simulation on a microcomputer. Current research involves testing the proposed algorithm on a real industrial system.

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Appendix 1

Referring to fig. 1 and the corresponding notation, each part's dynamics are considered separately and then combined to produce the overall model. In the present model, the fluid density and pipe diameter are considered to be constant throughout the system.

• For the pipe between the points 1 and 2 it holds that:

$$P_1 - P_2 = \frac{l_{12}}{A}\frac{\partial M_1}{\partial t} + \frac{\lambda_{12} l_{12}}{2\rho D A^2} M_1^2 \qquad (A.1)$$

where P_i is the total pressure at point i and λ_{ij} the friction coefficient for the part of pipe between points i and j.

• For the valve 2, 3 it holds that:

$$P_2 - P_3 = \frac{\xi_1}{2\rho A^2} M_1^2 \qquad (A.2)$$

• For the pipe between the points 3 and 4, where the leak is encountered, there are two models: one for the part of the pipe before the leak and one for the part after it. For the former it holds that:

$$P_3 - P_L = \frac{l_{3L}}{A}\frac{\partial M_1}{\partial t} + \frac{\lambda_{3L} l_{3L}}{2\rho D A^2} M_1^2 \qquad (A.3)$$

while for the latter:

$$P_L - P_4 = \frac{l_{L4}}{A}\frac{\partial M_2}{\partial t} + \frac{\lambda_{L4} l_{L4}}{2\rho D A^2} M_2^2 \qquad (A.4)$$

$$M_1 = M_2 + M_l \qquad (A.5)$$

$$M_1 = \sigma M_1 \qquad (A.6)$$

• For the valve 4, 5 it holds that:

$$P_4 - P_5 = \frac{\xi_2}{2\rho A^2} M_2^2$$

• For the pipe between the points 5 and 6 it holds that:

$$P_5 - P_6 = \frac{l_{56}}{A}\frac{\partial M_2}{\partial t} + \frac{\lambda_{56} l_{56}}{2\rho d A^2} M_2^2 \qquad (A.8)$$

• For the pump it holds that:

$$P_1 - P_0 = f(M_1, \omega) = \Delta P \qquad (A.9)$$

If it is assumed that the leak is small, the friction coefficients λ_{ij} can be considered equal to a single λ. Furthermore, if the system discharges to a tank with fluid in the same pressure P_0 as the feeding tank (which will be the case if both tanks are open or the fluid returns to the initial tank), then:

$$P_6 = P_0 + \frac{M_2^2}{2\rho A^2} \qquad (A.10)$$

Using (A.1)-(A.10) gives:

$$\frac{\partial m_1}{\partial t} = -\frac{\xi_1 + \frac{\lambda l_{1L}}{D} + \left(1 + \xi_2 + \frac{\lambda l_{L6}}{D}\right)(1-\sigma)^2}{2\rho\left(\sigma l_{1L} + l(1-\sigma)\right)A} M_1^2$$

$$+ \frac{A}{\sigma l_{1L} + l(1-\sigma)} f(M_1, \omega) \qquad (A.11)$$

Equation (A.11) is the system's state equation and describes the dynamic behaviour of the system. When the system operates in steady state, (A.11) becomes:

$$f(M_1,\omega) = -\frac{\xi_1 + \frac{\lambda l_{1L}}{D} + \left(1+\xi_2+\frac{\lambda l_{L6}}{D}\right)(1-\sigma)^2}{2\rho A^2} M_1^2 \tag{A.12}$$

$$= C M_1^2 \tag{A.13}$$

To obtain the initial values used in the numerical solution of (A.11), the input function described by (4) is used, together with (A.12), giving:

$$M_1(t_0) = \frac{h_2\omega(t_0) + \left[(h_2\omega(t_0))^2 + 4(C-h_1)h_3\omega^2(t_0)\right]^{1/2}}{2(C-h_1)} \tag{A.14}$$

Appendix 2

Taking the weighted sum of n_f input-output data points gives for (5a):

$$\frac{1}{n_f}\sum_{i=k}^{k+n_f-1} y(i) = \frac{1}{n_f}\sum_{i=k}^{k+n_f-1} \frac{\partial}{\partial t}\left(M_1(i)+v_M(i)\right)$$

$$= \frac{\partial}{\partial t}\sum_{i=k}^{k+n_f-1}\frac{1}{n_f}M_1(i) + \frac{\partial}{\partial t}\left(\frac{1}{n_f}\sum_{i=k}^{k+n_f-1} v_M(i)\right)$$

$$= \frac{\partial}{\partial t}\sum_{i=k}^{k+n_f-1}\frac{1}{n_f}M_1(i)$$

The RHS of (9) will consist of the following terms:

1)

$$\frac{1}{n_f}\sum_{i=k}^{k+n_f-1}\left[M_1^2(i) + v_M^2(i) + 2M_1(i)v_m(i)\right]$$

$$= \frac{1}{n_f}\left(\sum_{i=k}^{k+n_f-1}\left[M_1^2(i) + 2M_1(i)v_M(i)\right]\right) + \hat{s}_M^2$$

2)

$$\frac{1}{n_f}\sum_{i=k}^{k+n_f-1}\left[\omega^2(i) + v_\omega^2(i) + 2\omega(i)v_\omega(i)\right]$$

$$= \frac{1}{n_f}\left(\sum_{i=k}^{k+n_f-1}\left[\omega^2(i) + 2\omega(i)v_\omega(i)\right]\right) + \hat{s}_\omega^2$$

3)

$$\frac{1}{n_f}\sum_{i=k}^{k+n_f-1}\left[\omega(i)M_1(i) + v_\omega v_M(i) + \omega(i)v_M(i)\right.$$

$$\left. + M_1(i)v_\omega(i)\right] =$$

$$= \frac{1}{n_f}\left(\sum_{i=k}^{k+n_f-1}\left[\omega(i)M_1(i) + \omega(i)v_M(i) + M_1(i)v_\omega(i)\right]\right)$$

$$+ c\hat{o}v(v_M, v_\omega)$$

Collecting together all the terms, we get:

$$\frac{1}{n_f}\sum_{i=k}^{k+n_f-1}\frac{\partial}{\partial t}M_1(i) =$$

$$\frac{1}{n_f}\sum_{i=k}^{k+n_f-1}\left[M_1^2(i)\omega(i)M_1(i)\omega^2(i)\right]\theta +$$

$$+\frac{1}{n_f}\sum_{i=k}^{k+n_f-1}\left[2M_1(i)v_M(i)\omega(i)v_M(i)\right.$$

$$\left. + M_1(i)v_\omega(i)2\omega(i)v_\omega(i)\right]\theta +$$

$$+ c_n\lfloor \hat{s}_M^2 c\hat{o}v(v_M, v_\omega)\hat{s}_\omega^2\rfloor\theta$$

where $c_n = \frac{n_f}{n_f-1}$.

The first of the three terms of the RHS is similar to the system equation (5a). The second term consists of terms that are not usually zero, unless the values of $M_1(i)$ and $\omega(i)$ are fairly constant over the filtering interval of n_f values. Hence, if we let $M_1(i) = M_1$ and $\omega(i) = \omega$:

$$\frac{1}{n_f}\sum_{i=k}^{k+n_f-1}\left[2M_1(i)v_M(i)\omega(i)v_M(i)\right.$$

$$\left. + M_1(i)v_\omega(i)2\omega(i)v_\omega(i)\right] =$$

$$\left[\frac{2M_1}{n_f}\sum_{i=k}^{k+n_f-1} v_M(i)\vdots\frac{\omega}{n_f}\sum_{i=k}^{k+n_f-1} v_M(i)\right.$$

$$\left. + \frac{M_1}{n_f}\sum_{i=k}^{k+n_f-1} v_\omega(i)\vdots\frac{2\omega}{n_f}\sum_{i=k}^{k+n_f-1} v_\omega(i)\right] = 0$$

The third term consists of known values, and furthermore since the noise measurements are independent:

$$cov(v_M, v_\omega) = 0$$

Summing up, if noisy measurements are collected and averaged over n_f values, then the original parameter vector θ can be estimated from the equation:

$$\frac{1}{n_f}\sum_{i=k}^{k+n_f-1}\frac{\partial}{\partial t}M_1(i) =$$

$$= \left(\frac{1}{n_f} \sum_{i=k}^{k+n_f-1} \left[M_1^2(i)\omega(i) M_1(i)\omega^2(i) \right] - c_n \left[\hat{s}_M^2 0 \hat{s}_\omega^2 \right] \right) \mathbf{0}$$

Rearranging (5b) produces:

$$1 - \sigma = \sigma' = \frac{M_6(k) + w_M(k)}{M_1(k) + v_M(k)} \qquad (A.15)$$

Operating on (A.15) as for (5a) yields:

$$\frac{1}{n_f} \sum_{i=k}^{k+n_f-1} \sigma' = \frac{\frac{1}{n_f} \sum_{i=k}^{k+n_f-1} \left(M_6(i) + w_M(i) \right)}{\frac{1}{n_f} \sum_{i=k}^{k+n_f-1} \left(M_6(i) + w_M(i) \right)}$$

or:

$$\sigma' = \frac{\frac{1}{n_f} \sum_{i=k}^{k+n_f-1} M_6(i)}{\frac{1}{n_f} \sum_{i=k}^{k+n_f-1} M_1(i)}$$

since the noises are zero mean. Now (A.16) cannot be used to estimate σ unless, as before, the M_1 and M_6 are fairly constant in the pre-filter window. Under this restriction, (A.16) will filter out the noise and produce satisfactory estimates for σ.

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Figure Captions

Figure 1. Schematic diagram of simulated pumping system.

Figure 2. Variation of $\partial M / \partial t$ (simulated and calculated).

Figure 3. Leak position estimate, no measurement noise, $\mathrm{var}(e_1(k)) = (0.005)^2$ and $\mathrm{var}(e_2(k)) = (0.05)^2$.

Figure 4. Leak size (%) estimate, no measurement noise, $\mathrm{var}(e_1(k)) = (0.005)^2$ and $\mathrm{var}(e_2(k)) = (0.05)^2$.

Figure 5. Leak position estimate, $n_f = 30$, no modelling noise, $\mathrm{var}(v_M(k)) = (0.001)^2$ and $\mathrm{var}(v_\omega(k)) = (0.01)^2$.

Figure 6. Leak size estimate (%), $n_f = 30$, no modelling noise, $\mathrm{var}(v_M(k)) = (0.001)^2$ and $\mathrm{var}(v_\omega(k)) = (0.01)^2$.