Modeling of Active Vibration Control in Smart Structures

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***Abstract*- The paper presents active control of smart structures within a focused frame of piezoelectric applications in active vibration and noise attenuation with potential applications in mechanical and civil engineering. Smart structure has become an increasingly common term describing a structure embedded or bonded with a large number of lightweight active electro-mechanical sensors and actuators. In this paper, we consider the modelling and control issues related to smart structures bonded with piezoelectric sensors and actuators with emphasis on robust control design taking into account structural uncertainties using the H Infinity control theory. The results show that the vibrations can be significantly suppressed by H Infinity controller.**

Keywords- Smart Structures; Piezoelectric Composite; H Infinity Control

I. INTRODUCTION

Active structures have been intensively investigated in recent years ([2], [5], [6], [8], [10], [15], [17]). Due to their complexity, demands on performances and realisation forms, this topic represents an important interdisciplinary field of research with great application potentials. Smart structures and systems in general comprise integrity (structural and functional) of a structure or a system, multifunctional materials, actuators/sensors and appropriate control in order to achieve desired performances under varying environmental conditions ([6], [7], [8], [12]). The interdisciplinary field encompasses material science, applied mechanics (vibration, acoustics, fracture mechanics, elasticity), electronics (actuators, sensors, control), manufacturing, biomechanics etc.([1], [2], [9], [14], [15]) In this paper, an overall approach to active control of piezoelectric structures, which involves steps of modelling, control and simulation, is presented.

A composite beam laminated with piezoelectric sensors and actuators and subjected to external loads is considered in this paper. The governing equation of the beam is formulated. Finite elements are used to approximate the dynamics of the beam. The vibration control problem is stated in terms of disturbance attenuation requirement for external disturbances. H Infinity control strategy is applied to solve the posed problem.

Considering the uncertainty, which arises for example from neglecting higher order dynamics, insufficient knowledge of the real plant parameters, external disturbances and measure noise, an H Infinity robust controller is designed. The advantage of the H Infinity criterion is its ability to take into account the worst influence of uncertain disturbances or noise in the system. It is possible to synthesize a H Infinity controller, which will be robust with respect to a predefined level of deviation from a nominal (ideal) model.

II. MATHEMATICAL MODELLING

A cantilever slender beam with rectangular cross-sections is considered. Four pairs of piezoelectric patches are embedded symmetrically at the top and the bottom surfaces of the beam, as shown in Fig.1.

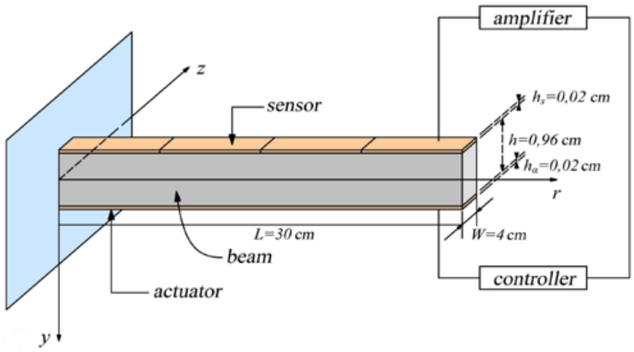

Fig. 1 Smart beam

The beam is from graphite- epoxy T300 − 976 and the piezoelectric patches are PZT G1195N. The top patches act like sensors and the bottom like actuators. The resulting composite beam is modelled by means of the classical laminated technical theory of bending. Let us assume that the mechanical properties of both the piezoelectric material and the host beam are independent in time. The thermal effects are considered to be negligible as well [13].

The beam has length L, width W and thickness h. The sensors and the actuators have width bS and bA and thickness hS and hA, respectively. The electromechanical parameters of the beam of interest are given in the Table Ⅰ.

TABLE I PARAMETERS OF THE COMPOSITE

Parameters	**Values**
Beam Length, L	0.3m
Beam Width, W	0.04m
Beam Thickness, H	0.0096m
Beam Density, P	1600kg/m^3
Youngs Modulus of the Beam, E	1.5 X 10^{11} N/m^2
Piezoelectric Constant, d_{31}	254 X 10^{-12} m/V
Electric Constant, ξ_{33}	11.5 X 10^{-3} V m/N
Young's Modulus of the Piezoelectric Element	1.5 X 10^{11} N/m^2
Width of the Piezoelectric Element	b_S= b_a= 0.04m
Thickness of the Piezoelectric Element	h_S= h_a= 0.0002m

A. Reduced Piezoelectric Equations

In order to derive the basic equations for piezoelectric sensors and actuators (S/As), we assume that:

- The piezoelectric S/A are bonded perfectly on the host beam;
- The piezoelectric layers are much thinner then the host beam;
- The piezoelectric material is homogeneous, transversely isotropic and linearly elastic;
- The piezoelectric S/A are transversely polarized (in the z-direction) [13].

Under these assumptions the three-dimensional linear constitutive equations are given by [6],

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{xz} \end{Bmatrix} - \begin{bmatrix} d_{31} \\ 0 \end{bmatrix} E_z \right) \tag{1}$$

$$D_z = Q_{11} d_{31} \varepsilon_{xx} + \xi_{xx} E_z \tag{2}$$

where σ_{xx} , σ_{xz} denote the axial and shear stress components, D_z , denotes the transverse electrical displacement; ε_{xx} and ε_{xz} are axial and shear strain components; Q_{11}, and Q_{55}, denote elastic constants; d_{31}, and ξ_{33}, denote piezoelectric and dielectric constants, respectively. Equation 1 describes the inverse piezoelectric effect and Equation 2 describes the direct piezoelectric effect. E_z, is the transverse component of the electric filed that is assumed to be constant for the piezoelectric layers and its components in xy-plain are supposed to vanish. If no electric filed is applied in the sensor layer, the direct piezoelectric Equation 2 gets the form,

$$D_z = Q_{11} d_{31} \varepsilon_{xx} \tag{3}$$

and it is used to calculate the output charge created by the strains in the beam [6].

B. Equations of Motion

The length, width, and thickness of the host beam are denoted by L, b and h. The thickness of the sensor and actuator is denoted by hS and hA. We assume that:

- The beam centroidal and elastic axis coincides with the x-coordinate axis so that no bending-torsion coupling is considered;
- The axial vibration of the host beam centreline is considered negligible;
- The displacement field {u} = (u1, u2, u3) is obtained based on the usual Timoshenko assumptions[4],

$$\begin{gathered} u_1(x, y, z) \approx z\varphi(x, t) \\ u_2(x,y,z) \approx 0 \\ u_3(x, y, x) \approx \omega(x, t) \end{gathered} \tag{4}$$

where φ is the rotation of the beams cross-section about the positive y-axis and w is the transverse displacement of a point of the centroidal axis (y = z = 0).

The strain displacement relations can be applied to Equation 4 to give,

$$\varepsilon_{xx} = z\frac{\partial \varphi}{\partial x}, \varepsilon_{xz} = \frac{\partial \omega}{\partial x} \tag{5}$$

We suppose that the transverse shear deformation ε_{xx} is equal to zero [4].

In order to derive the equations of the motion of the beam we use Hamilton's principle,

$$\int_{t_2}^{t_1} \left(\delta \mathrm{T} - \delta U + \delta W\right) dt = 0 \tag{6}$$

where T [14] is the total kinetic energy of the system, U is the potential (strain) energy and W is the virtual work done by the external mechanical and electrical loads and moments. The first variation of the kinetic energy is given by,

$$\begin{aligned} \delta T &= \frac{1}{2}\int_V \rho \left\{\frac{\partial u}{\partial t}\right\}^r \left\{\frac{\partial u}{\partial t}\right\} dV \\ &= \frac{b}{2}\int_0^L \int_{-\frac{h}{2}-h_A}^{\frac{h}{2}+h_s} \rho(z\frac{\partial \varphi}{\partial t}\delta\frac{\partial \varphi}{\partial t} + \frac{\partial \omega}{\partial t}\delta\frac{\partial \omega}{\partial t}) dzdx \end{aligned} \tag{7}$$

The first variation of the potential energy is given by,

$$\begin{aligned} \delta U &= \frac{1}{2}\int_V \delta\{\varepsilon\}^{\mathrm{T}}\{\sigma\} dV \\ &= \frac{b}{2}\int_0^L \int_{-\frac{h}{2}-h_A}^{\frac{h}{2}+h_s} \left[Q_{11}\left(z\frac{\partial \omega}{\partial x}\delta\right)\left(z\frac{\partial \omega}{\partial x}\right)\right] dzdx \end{aligned} \tag{8}$$

If the load consists only of moments induced by piezoelectric actuators and since the structure has no bending twisting couple then the first variation of the work has the form [14],

$$\delta W = b\int_0^L M^A \delta\left(\frac{\partial \varphi}{\partial x}\right) dx \tag{9}$$

where M^A is the moment per unit length induced by the actuator layer and is given by,

$$M^A = \int_{-\frac{h}{2}-h_A}^{-\frac{h}{2}} z\sigma_{xx}^{A} dz = \int_{-\frac{h}{2}-h_A}^{-\frac{h}{2}} zQ_{11}d_{31}E_z^A dz (E_z^A = \frac{V_A}{h_A}) \tag{10}$$

C. Finite Element Formulation

We consider a beam element of length Le has two mechanical degrees of freedom at each node: one translational ω1 (respectively ω2) in direction z and one rotational φ1 (respectively φ2), as it is shown in Fig. 2.

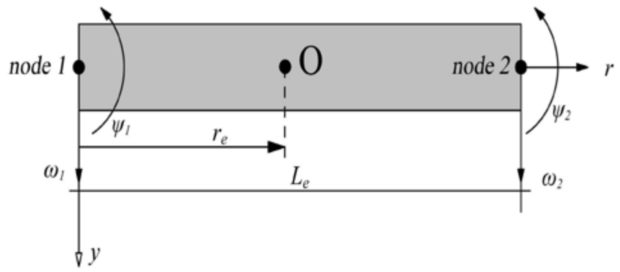

Fig. 2 Beam finite element

The vector of nodal displacements and rotations q_e is defined as [5-7, 13],

$$q_e^r = \left[\omega_1, \psi_1, \omega_2, \psi_2\right] \tag{11}$$

The beam element transverse deflection ω(x, t) and the beam element rotation ψ(x, t) of the beam are continuous and they are interpolated within by Hermitian linear shape functions H_i^{ω} and H_i^{ψ} as follows,

$$\omega(x,t) = \sum_{i=1}^{4} H_i^{\omega}(x) q_i(t)$$
$$\psi(x,t) = \sum_{i=1}^{4} H_i^{\psi}(x) q_i(t) \qquad (12)$$

This classical finite element procedure leads to the approximate (discretized) variational problem. For a finite element the discrete differential equations are obtained by substituting the discretized Expressions 12 into Equations 7 and 8 to evaluate the kinetic and strain energies. Integrating over spatial domains and using the Hamilton's Principle 6, the equation of motion for a beam element are expressed in terms of nodal variable q as follows,

$$M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = f_m(t) + f_e(t) \qquad (13)$$

where *M* is the generalized mass matrix, *D* the viscous damping matrix, *K* the generalized stiffness matrix, the external loading vector and the generalized control force vector produced by electromechanical coupling effects. The independent variable q(t) is composed of transversal deflections and rotations , i.e., [18]

$$q(t) = \begin{bmatrix} \omega_1 \\ \psi_1 \\ \vdots \\ \omega_n \\ \psi_n \end{bmatrix} \qquad (14)$$

Where n is the number of nodes used in analysis. Equation 13 is transformed into the usual state-space control form,

$$\dot{x}(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \qquad (15)$$

Furthermore to express $f_e(t)$ as $Bu(t)$ we write it as $f_e^* u$ where f_e^* is the piezoelectric force for a unit applied on the corresponding actuator, and u represents the voltages on the actuators. Furthermore, $d(t) = f_m(t)$ is the disturbance vector [12].

Then,

$$\dot{x}(t) = \begin{bmatrix} O_{2n\times 2n} & I_{2n\times 2n} \\ -M^{-1}K & -M^{-1}D \end{bmatrix} x(t) + \begin{bmatrix} O_{2n\times 2n} \\ M^{-1} f_e^* \end{bmatrix} u(t) + \begin{bmatrix} O_{2n\times 2n} \\ M^{-1} \end{bmatrix} \qquad (16)$$

$$= Ax(t) + Bu(t) + Gd(t) = Ax(t) + \begin{bmatrix} B & G \end{bmatrix} \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} = Ax(t) + \tilde{B}\tilde{u}(t) \qquad (17)$$

The previous description of the dynamical system will be augmented with the output equation (displacements only measured) [6],

$$y(t) = \begin{bmatrix} x_1(t) & x_3(t) & \cdots & x_{n-1}(t) \end{bmatrix}^T = Cx(t) \qquad (18)$$

In this formulation u is n x 1 (at most, but can be smaller), while d is 2n x 1. The units used are compatible for instance m, rad, sec and N.

III. DESIGN OBJECTIVES AND SYSTEM SPECIFICATIONS

The structured singular value of a transfer function matrix is defined as,

$$\mu(M) = \begin{cases} \dfrac{1}{\min_{k_m}\{\det(I - k_m M\Delta) = 0,\ \bar{\sigma}(\Delta) \le 1\}} \\ 0, \text{if no such structured exists} \end{cases} \qquad (19)$$

In words it defines the smallest structured Δ (measured in terms of the nominal values of the system) which makes det (I−MΔ)=0: then $\mu(M) = 1/\bar{\sigma}(\Delta)$. It follows those values of μ smaller than 1 are desired (the smaller the better: a larger variation is allowed).

A. *Design Objectives*

Design objectives fall into two categories:

Nominal performance

1. Stability of closed loop system (plant+controller);
2. Disturbance attenuation with satisfactory transient characteristics (overshoot, settling time);
3. Small control effort.

Robust performance

4. (1)-(3) above should be satisfied in the face of modelling errors.

B. *System Specifications*

To obtain the required system specifications to meet the above objectives we need to represent our system in the so-called (N, Δ) structure. In order to achieve this, let us start with the simple typical diagram of Fig. 3.

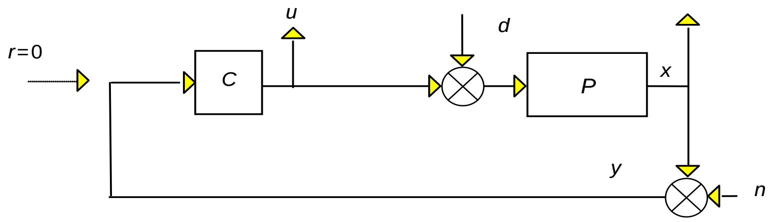

Fig. 3 Classical control block diagram

In this diagram there are two inputs, d (disturbances) and n (noise), and two outputs, u (control vector) and x (state vector). In what follows, it is assumed that,

$$\left\| \begin{matrix} d \\ n \end{matrix} \right\|_2 \leq 1, \quad \left\| \begin{matrix} u \\ x \end{matrix} \right\|_2 \leq 1 \tag{20}$$

If that is not the case, appropriate frequency-dependent weights can transform original signals so that the transformed signals have this property.

After rewriting Fig. 3 in frequency domain, one gets the system in Fig. 4:

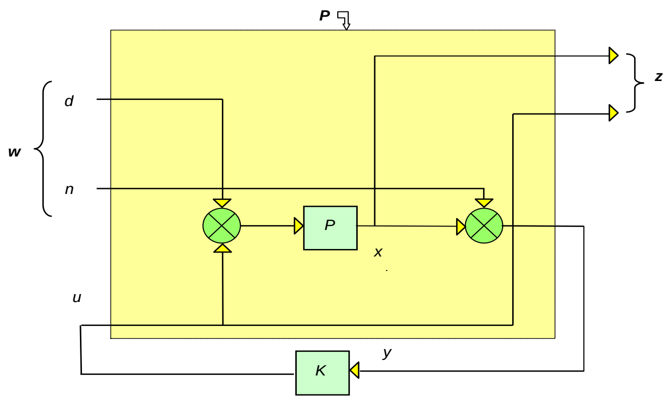

Fig. 4 Detailed two-port diagram

or in less detailed Fig. 5,

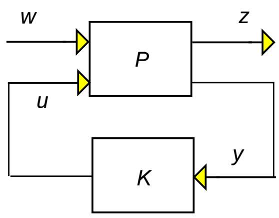

Fig. 5 Two-port diagram

with,

$$z = \begin{bmatrix} u \\ x \end{bmatrix}, \quad w = \begin{bmatrix} d \\ n \end{bmatrix} \tag{21}$$

where z are the output variables to be controlled, and w the exogenous inputs.

Given that P has two inputs and two outputs, it is, as usual, naturally partitioned as,

$$\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} P_{zw}(s) & P_{zu}(s) \\ P_{yw}(s) & P_{yu}(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \overset{\text{op}}{=} P(s) \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \tag{22}$$

Also,

$$u(s)=K(s)y(s) \tag{23}$$

Substituting (23) in (22) gives the closed loop transfer function $N_{zw}(s)$,

$$N_{zw}(s)=P_{zw}(s)+P_{zu}(s)K(s)(I-P_{yu}(s)K(s))^{-1}P_{yw}(s) \tag{24}$$

IV. CONTROLLER SYNTHESIS

It is possible to approximately synthesize a controller that achieves given performance in terms of the structured singular value μ.

In this procedure known as (D, G-K) Iteration (14) the problem of finding a μ-optimal controller K such that μ(Φu(F(jω), K(jω))$\leqslant \beta$, ∀ω, is transformed into the problem of finding transfer function matrices D(ω)$\in \Delta$ and G(ω) $\in \Gamma$, such that,

$$\sup_{\omega} \bar{\sigma}\left[\left(\frac{D(\omega)(\square_u(F(j\omega),\mathrm{K}(j\omega))D^{-1}(\omega)}{\gamma} - \mathrm{j}G(\omega)\right)\left(\mathrm{I}+G^2(\omega)\right)^{-\frac{1}{2}}\right] \le 1, \forall \omega \tag{25}$$

Unfortunately this method does not guarantee even finding local maxima. However for complex perturbations a method known as D-K iteration is available (also implemented in MATLAB), [11, 12, 18]. It combines H$_\infty$ synthesis and μ-analysis and often yields good results. The starting point is an upper bound on μ in terms of the scaled singular value,

$$\mu(N) \le \min_{D \in \square} \bar{\sigma}(DND^{-1}) \tag{26}$$

The idea is to find the controller that minimizes the peak over frequency of its upper bound, namely,

$$\min_{K}\left(\min_{D \in \square} \left\| DN(K)D^{-1} \right\|_\infty\right) \tag{27}$$

by alternating between minimizing $\left\| DN(K)D^{-1} \right\|_\infty$ with respect to either K or D (while holding the other fixed).[11]

1. **K-step**. Synthesize an H$_\infty$ controller for the scaled problem $\min_K \left\| DN(K)D^{-1} \right\|_\infty$ with fixed $D(s)$.
2. **D-step**. Find $D(\mathrm{j}\omega)$ to minimize at each frequency $\bar{\sigma}(DND^{-1}(\mathrm{j}\omega))$ with fixed N.
3. Fit the magnitude of each element of $D(\mathrm{j}\omega)$ to a stable and minimum phase transfer function $D(s)$ and go to Step 1.

V. RELATION TO THE BEAM CONTROL PROBLEM

Our aim is to find the appropriate N matrix, as defined in (24). To this aim it is useful, to derive the input-output relations for the original model,

$$\begin{bmatrix} u \\ e \end{bmatrix} = F(s) \begin{bmatrix} d \\ n \end{bmatrix} \Rightarrow z = F(s)w \tag{28}$$

as depicted in Fig. 6.

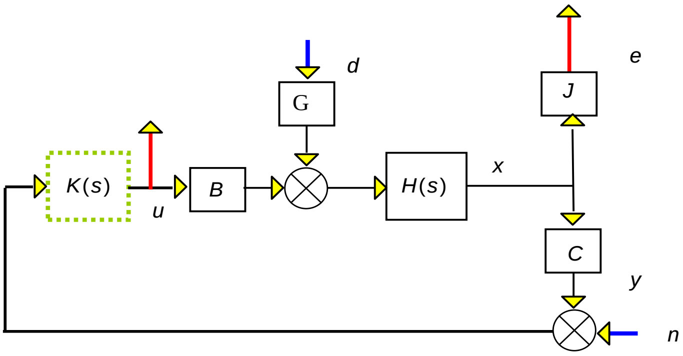

Fig. 6 Beam with controller

where the beam is described by,

$$\dot{\mathbf{x}}(t) = \boldsymbol{A}\mathbf{x}(t) + [\boldsymbol{B}\ \boldsymbol{G}\ ]\begin{bmatrix}\mathbf{u}(t)\\ \mathbf{d}(t)\end{bmatrix} \tag{29}$$
$$\Rightarrow \boldsymbol{H}(s) = (s\boldsymbol{I} - \boldsymbol{A})^{-1}$$

and J is used to pick up those states that we are interested in regulating (may be different from y). In most of the experiments, J will be,

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{30}$$

To continue, we use appropriate weighting and redraw Fig.6 to fit our problem:

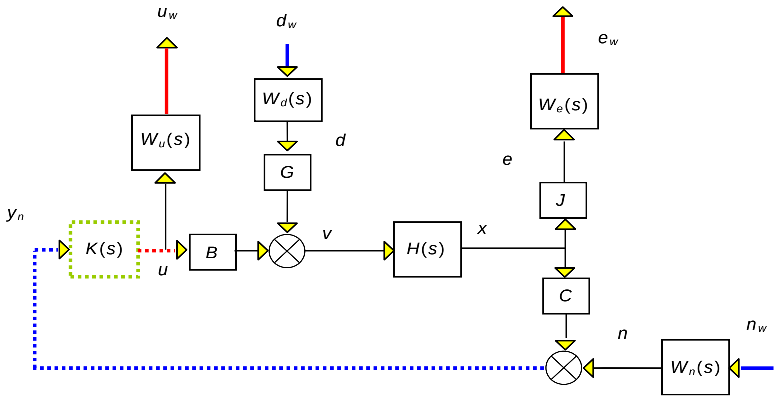

Fig. 7 Weighted block diagram for the beam problem

Then redraw Fig. 7 as a two port diagram, similar to Fig.8:

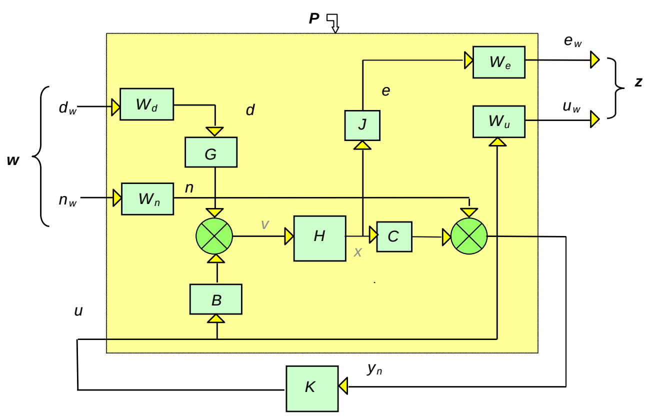

Fig. 8 Two port diagram for the beam problem

In Fig. 8, x, v are auxiliary signals.

We are looking for,

$$Q_{zw}(s)=P_{zw}(s)+P_{zu}(s)K(s)(I-P_{yu}(s)K(s))^{-1}P_{yw}(s) \tag{31}$$

such that,

$$z=Q_{zw}w=\Phi_l(P,K)w \tag{32}$$

We need to find $P(s)$. The necessary transfer functions are,

$$\begin{aligned} e_w &= W_e Jx = W_e JHv = W_e JH(GW_d d_w + Bu) \\ &= W_e JHGW_d d_w + W_e JHBu \end{aligned} \tag{33}$$

$$u_w = W_u u \tag{34}$$

$$\begin{aligned} y_n &= Cx + W_n n_w = CHv + W_n n_w = CH(GW_d d_w + Bu) + W_n n_w \\ &= CHGW_d d_w + CHBu + W_n n_w \end{aligned} \tag{35}$$

Combining all these gives,

$$\begin{bmatrix} u_w \\ e_w \\ \hline y_n \end{bmatrix} = \left[\begin{array}{cc|c} 0 & 0 & W_u \\ W_e JHGW_d & 0 & W_e JHB \\ \hline CHGW_d & W_n & CHB \end{array}\right] \begin{bmatrix} d_w \\ n_w \\ \hline u \end{bmatrix} \tag{36}$$

or,

$$\begin{bmatrix} z \\ y_n \end{bmatrix} = \begin{bmatrix} P_{zw} & P_{zu} \\ P_{yw} & P_{yu} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{37}$$

where,

$$\begin{aligned} &P_{zw} = \begin{bmatrix} 0 & 0 \\ W_e JHGW_d & 0 \end{bmatrix}, \; P_{zu} = \begin{bmatrix} W_u \\ W_e JHB \end{bmatrix}, \\ &P_{yw} = \begin{bmatrix} CHGW_d & W_n \end{bmatrix}, \; P_{yu} = CHB \end{aligned} \tag{38}$$

One more step is needed, however to get the Q_{ij}'s. We do this using

$$\begin{bmatrix} u \\ e \end{bmatrix} = \begin{bmatrix} (I-KCHB)^{-1}KCHG & (I-KCHB)^{-1}K \\ J(I-HBKC)^{-1}HG & J(I-HBKC)^{-1}HBK \end{bmatrix} \begin{bmatrix} d \\ n \end{bmatrix} \tag{39}$$

and noting that,

$$d=W_d d_w,\; n=W_n n_w,\; e_w=W_e e,\; u_w=W_u u$$

Hence,

$$\begin{aligned} &\begin{bmatrix} u \\ e \end{bmatrix} = \begin{bmatrix} W_u^{-1}u_w \\ W_e^{-1}e_w \end{bmatrix} = F(s)\begin{bmatrix} d \\ n \end{bmatrix} = F(s)\begin{bmatrix} W_d d_w \\ W_n n_w \end{bmatrix} \Rightarrow \\ &\begin{bmatrix} u_w \\ e_w \end{bmatrix} = \begin{bmatrix} W_u & \\ & W_e \end{bmatrix} F(s) \begin{bmatrix} W_d & \\ & W_n \end{bmatrix} \begin{bmatrix} d_w \\ n_w \end{bmatrix} \end{aligned} \tag{40}$$

or,

$$\begin{bmatrix} u_w \\ e_w \end{bmatrix} = \begin{bmatrix} W_u(I-KCHB)^{-1}KCHGW_d & W_u(I-KCHB)^{-1}KW_n \\ W_e J(I-HBKC)^{-1}HGW_d & W_e J(I-HBKC)^{-1}HBKW_n \end{bmatrix} \begin{bmatrix} d_w \\ n_w \end{bmatrix} \tag{41}$$

The compact expression in matrix form reads,

$$z = Q_{zw}w \text{ or } \begin{bmatrix} u \\ e \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} d \\ n \end{bmatrix} \tag{42}$$

To express P in state space, let us first form the natural partitioning,

$$P(s)=\left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]=\begin{bmatrix} P_{zw}(s) & P_{zu}(s) \\ P_{yw}(s) & P_{yu}(s) \end{bmatrix} \tag{43}$$

(where the packed form has been used), while the corresponding form for K is [3],

$$K(s)=\left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array}\right]$$

Eq. (43) defines the equations,

$$\begin{aligned} \dot{x}(t) &= Ax(t)+\begin{bmatrix} B_1 & B_2 \end{bmatrix}\begin{bmatrix} w(t) \\ u(t) \end{bmatrix} \\ \begin{bmatrix} z(t) \\ y(t) \end{bmatrix} &= \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}x(t)+\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}\begin{bmatrix} w(t) \\ u(t) \end{bmatrix} \end{aligned} \tag{44}$$

and,

$$\dot{x}_K(t)=A_K x_K(t)+B_K y(t) \tag{45}$$

$$u(t)=C_K x_K(t)+D_K y(t) \tag{46}$$

To get the structure in state space form, relate the inputs, outputs, states, and input/output to the controller (Fig. 9):

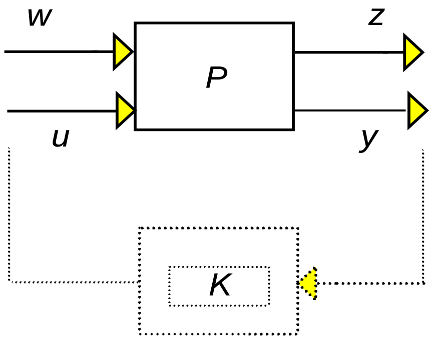

Fig. 9 Open loop structure

To find the matrices involved, we break the feedback loop and use the relevant equations:

$$\begin{aligned} &\dot{x}_F = Ax_F + (Gd+Bu), && x = Ix_F \\ &\dot{x}_u = A_u x_u + B_u u, && u_w = C_u x_u + D_u u \\ &\dot{x}_e = A_e x_e + B_e Jx, && e_w = C_e x_e + D_e Jx \\ &\dot{x}_{nw} = A_{nw} x_{nw} + B_{nw} n_w, && n = C_{nw} x_{nw} + D_{nw} n_w \\ &\dot{x}_{dw} = A_{dw} x_{dw} + B_{dw} d_w, && d = C_{dw} x_{dw} + D_{dw} d_w \\ &y = Cx + n \end{aligned} \tag{47}$$

Let, $x=\begin{bmatrix} x_F \\ x_u \\ x_e \\ x_{nw} \\ x_{dw} \end{bmatrix}$, $y=y$, $w=\begin{bmatrix} d_w \\ n_w \end{bmatrix}$, $z=\begin{bmatrix} u_w \\ e_w \end{bmatrix}$, $u=u$

Substituting the internal signals d, n, e και x from (47), yields,

$$\begin{bmatrix} \dot{x}_F \\ \dot{x}_u \\ \dot{x}_e \\ \dot{x}_{nw} \\ \dot{x}_{dw} \end{bmatrix} = \begin{bmatrix} A & 0 & 0 & 0 & GC_{dw} \\ 0 & A_u & 0 & 0 & 0 \\ B_e J & 0 & A_e & 0 & 0 \\ 0 & 0 & 0 & A_{nw} & 0 \\ 0 & 0 & 0 & 0 & A_{dw} \end{bmatrix} \begin{bmatrix} x_F \\ x_u \\ x_e \\ x_{nw} \\ x_{dw} \end{bmatrix} + \begin{bmatrix} GD_{dw} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & B_{nw} \\ B_{dw} & 0 \end{bmatrix} \begin{bmatrix} d_w \\ n_w \end{bmatrix} + \begin{bmatrix} B \\ B_u \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$\begin{bmatrix} u_w \\ e_w \end{bmatrix} = \begin{bmatrix} 0 & C_u & 0 & 0 & 0 \\ D_e J & 0 & C_e & 0 & 0 \end{bmatrix} \begin{bmatrix} x_F \\ x_u \\ x_e \\ x_{nw} \\ x_{dw} \end{bmatrix} + 0 \begin{bmatrix} d_w \\ n_w \end{bmatrix} + \begin{bmatrix} D_u \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} C & 0 & 0 & C_{nw} & 0 \end{bmatrix} \begin{bmatrix} x_F \\ x_u \\ x_e \\ x_{nw} \\ x_{dw} \end{bmatrix} + \begin{bmatrix} 0 & D_{nw} \end{bmatrix} \begin{bmatrix} d_w \\ n_w \end{bmatrix} + 0u$$

Therefore the matrices are:

$$A_1 = \begin{bmatrix} A & 0 & 0 & 0 & GC_{dw} \\ 0 & A_u & 0 & 0 & 0 \\ B_e J & 0 & A_e & 0 & 0 \\ 0 & 0 & 0 & A_{nw} & 0 \\ 0 & 0 & 0 & 0 & A_{dw} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} GD_{dw} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & B_{nw} \\ B_{dw} & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} B \\ B_u \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & C_u & 0 & 0 & 0 \\ D_e J & 0 & C_e & 0 & 0 \end{bmatrix}, \quad D_{11} = 0, \quad D_{12} = \begin{bmatrix} D_u \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} C & 0 & 0 & C_{nw} & 0 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 0 & D_{nw} \end{bmatrix}, \quad D_{22} = 0$$

As can be seen, in this configuration the size of the state vector is 16+4+4+4+8=36. This will also be the size of the controller model $K(s)$. This number will be decreased, if some weight matrices are constant.

VI. RESULTS

A typical wind load (Fig. 10) acting on the side of the structure. The wind load is a real life wind speed measurements in relevance with time that took place in Estavromenos of Heraklion Crete. We transform the wind speed in wind pressure with,

$$p(t) = \frac{1}{2} \rho C_u V^2(t) \tag{48}$$

where V=velocity, ρ=density and C_u=1.5.

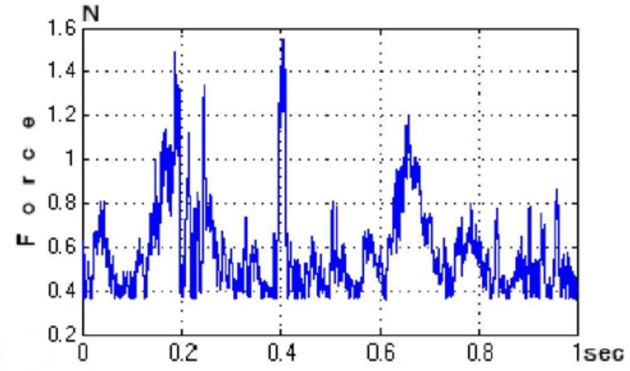

Fig. 10 Corresponding wind load acting.

In Fig. 11, the structural displacement responses are reported, for the four nodes of the beam. It is possible to see clearly the benefit induced the control on the maximum value of these displacements. In Fig. 12, we can see the structural rotations for the four nodes of the beam. The beam with H_∞ control keeps in equilibrium and we have almost zero displacements.

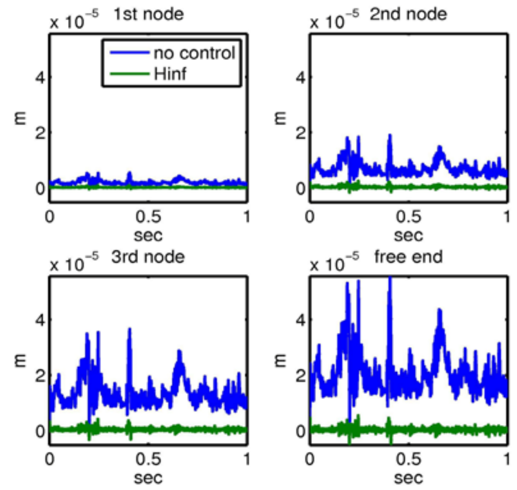

Fig. 11 Responses of the four nodes of the beam without and with H_∞ control

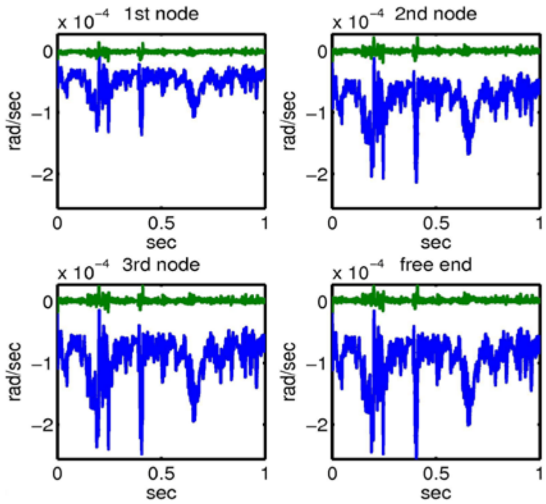

Fig. 12 Rotations of the four nodes of the beam without and with H_∞ control

In Fig. 13, we can see the control profile for the four nodes of the beams. As we can see the voltage is more less than 500Volt, which is considered in most cases to be the piezoelectric limit.

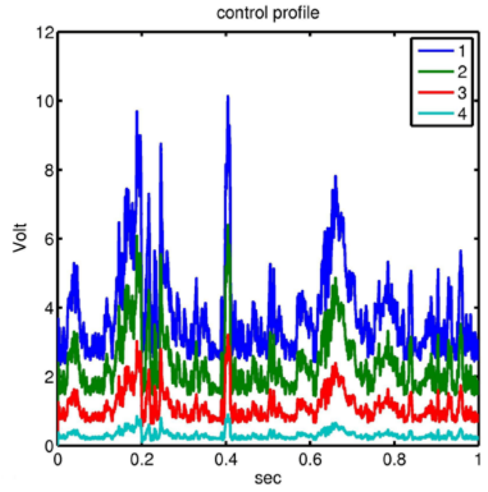

Fig. 13 Control voltages for the four nodes of the beam using H_∞ control

The controller obtained by applying H_∞ control has an order equal to 36. For this controller, $\gamma = 0.074$. A plot of the maximum singular value of the weighted closed-loop system (beam plus H_∞ controller) is given in Fig. 14, where we can clearly note that the value remains below γ at all frequencies.

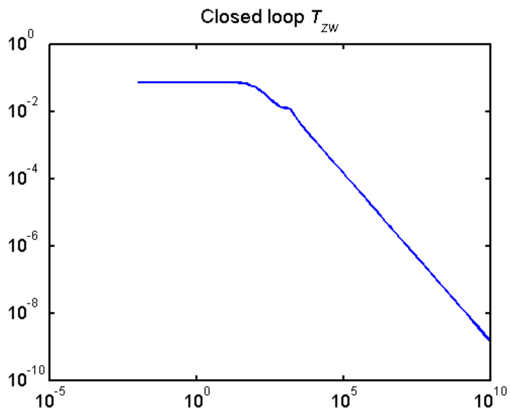

Fig. 14 Maximum singular value of the closed-loop system

Fig. 15 presents the Bode diagrams of diagonal elements of the above weight matrices. These matrices have been obtained through a number of tests, to ensure feasibility of finding a controller H_∞.

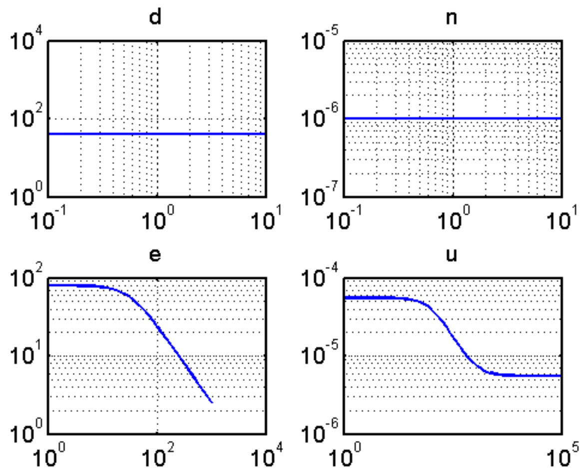

Fig. 15 Bode diagram

Figs. 16 a, b, further show the maximum singular values of transfer functions of the closed-loop system (i.e. the initial one).

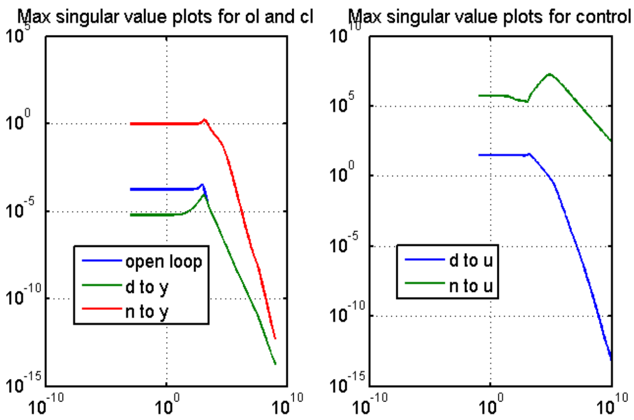

Fig. 16 a,b Max singular values

These figures show that the performance of the computed controller is satisfactory, since:

1) As shown in Fig. 16a, there is a significant improvement in the effect of disturbance on error up to the frequency of 1000 Hz.

2) As shown in Fig. 16a, there seems to be little effect of noise on error for frequencies beyond 1000 Hz.

3) Fig. 16b shows a satisfactory effect of the disturbance on the size of the control scheme (the design could be improved, if it were possible to reduce noise effect for frequencies of 1000 Hz).

VII. CONCLUSIONS

This paper presents a mathematical formulation and computational model for active vibration control of a slender beam bonded with piezoelectric sensors and actuators. The problem of active control is studied by using the H Infinity approach in order to achieve robustness with respect to external disturbances in the presence of structural uncertainties. Proper selection of the involved parameters is very important for a successful design of this controller. A suboptimal controller has been used for numerical modelling. The comparison between the responses of the uncontrolled and controlled beam using the proposed control law shows that the control strategy is effective. The results are very good: the oscillations were suppressed even for a real Aeolian load, with the voltages of the piezoelectric components laying within their endurance limits. The advantage of the H Infinity criterion is its ability to take into account the worst influence of uncertain disturbances or noise in the system. It is possible to synthesize H Infinity controller which will be robust with respect to a predefined level of structural uncertainty.

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