Detection of multivariable system noise degradation

Anastasios D. Pouliezos
Technical University of Crete

G. Stavrakakis
Technical University of Crete

Detection of multivariable system noise degradation

A. Pouliezos*, G. Stavrakakis** and G. Tselentis*

**Department of Production and Management Engineering*
***Department of Electronic and Computer Engineering*
Technical University of Crete, University Campus, Counoupidiana, 731 00 Chania, Greece
Voice: +(30 821) 69549, e-mail: tasos@hra.ergasya.tuc.gr

Received May 1995, revised September 1995
Communicated by Madan G. Singh

In this paper, a three-stage monitoring scheme is proposed and evaluated for sensor noise degradation in discrete linear stochastic systems. In the first stage, change detection is achieved, based on the effects that additional sensor noise has on the joint PDF of the Kalman–Bucy filter innovations. Change identification is achieved with the aid of the steady state (following the change) innovations covariance matrix. System reorganization is then realised on the basis of these findings. All necessary calculations are performed recursively, thus making the whole scheme suitable for on line diagnostic systems. The proposed method is illustrated and verified by computer simulation of a multivariable system using *FAULTLAB*, a fault simulation computer toolbox.

1. Introduction

The task of *Change Detection and Identification* (CDI) is one of the most important operations assigned to computers supervising controlled plants. This is a direct consequence of the growing complexity of both the plants and their controlling algorithms. CDI monitoring of system condition can help avoid degradation of system performance, since in most cases the control law must be reconfigured to reflect the changes. For this to be accomplished, CDI should perform the following tasks:

- change detection (alarm),
- change isolation (what part of the system has changed),
- change identification (size and, possibly, time of change),
- system reorganization (resetting of controller and CDI parameters).

Furthermore, the above tasks should be carried out in a robust way, that is modeling errors and noise should not trigger false alarms while the probability of correct detection should remain high. These are usually conflicting requirements, and some sort of compromise solution is usually implemented.

Change detection ideas have been applied to many industrial areas: chemical industry [9], nuclear industry [8], aeronautics [7], leak detection in pipelines [5], vibration monitoring [3]. Moreover, CDI plays an important part in the design of autonomous control systems [19]. Indeed, a well designed autonomous control system must incorporate some form of low and high level fault detection subsystem. Finally, CDI, if

IOS Press
Information and Systems Engineering 1 (1995) 255–270

0929-9610/95/$07.00

properly designed, will result in improved robustness of the overall system, since unexpected destabilizing changes can be detected and accounted for.

Fault detection methods fall into three main categories:

- model-based (parameter/state estimation),
- knowledge-based (expert and rule-based systems),
- artificial neural networks-based.

Each method possesses certain advantages and disadvantages, and it is also possible to combine approaches, resulting in hybrid schemes. For a presentation of these methods and their applications, consult [14].

In this paper, the problem of detecting increased sensor noise in stochastic systems whose controllers use Kalman filter estimates, is addressed. Since the optimal operation of the Kalman filter state estimator depends on the correctness of the system parameters, it is necessary that they are constantly monitored to check whether they remain in acceptable statistical limits, given their pre-estimated values. In particular, the sensor noise covariance, which enters into the filter calculations, affects the state estimate error covariance and thus if increased, produces suboptimal state estimates, i.e., estimates with larger error covariance (see Fig. 1 for illustration). Therefore, it is important to have a sensor noise covariance change detector, if the optimality of the Kalman filter is desired under every operating condition. The effect of jumps in observation noise statistics on the Kalman filter estimates, is investigated in some detail for the case of power systems control in [6].

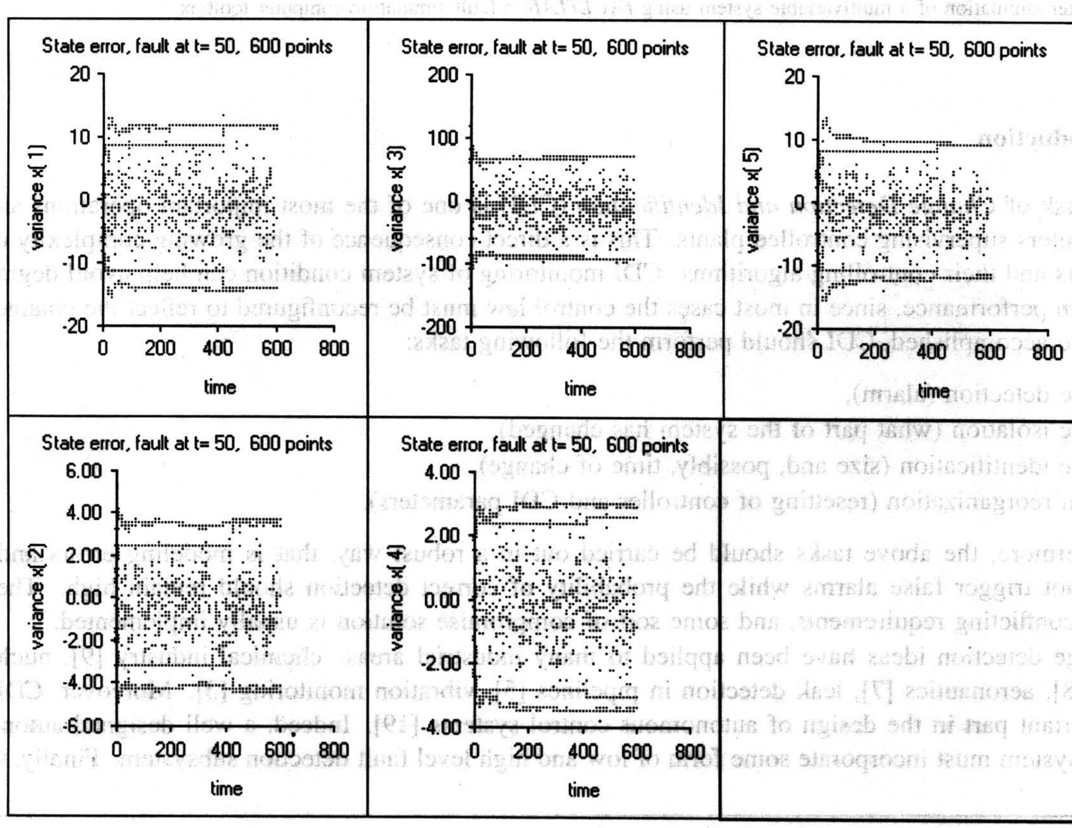

Fig. 1. Evolution of the state estimate error

The proposed methodology can be applied to two other circumstances as well:

- In cases where the sensor noise covariance is unknown and thus simultaneous state estimation and sensor noise covariance identification is needed. In this case, an initial estimate of the noise covariance is used and subsequently the change monitoring scheme "detects" the correct value.
- In cases where the sensor noise covariance is slowly changing (slower than the time needed for the change monitoring scheme to respond). In this case the proposed methodology can be used to "track" the time-varying covariance value.

Previous attempts for the solution of this problem have aimed only at the detection phase of the fault monitoring process: in [11], SPR (*Sequential Probability Ratio*) and GLR (*Generalised Likelihood Ratio*) are used for sensor noise degradation detection. Though GLR can, in theory, perform all required tasks, no practical implementation was presented. In [18], a backward SPRT (*Sequential Probability Ratio Test*) is proposed for the detection of the innovations variance change, in cases where the change magnitude is known.

In this paper, the approach of model-based CDI is adopted. A minor deficiency that this methodology shares with other model-based techniques, is the need for a precise mathematical model. However, this requirement is easily fulfilled using current model identification techniques which permit acceptable mathematical models to be developed for many industrial processes. Thus, in such applications, the proposed method can greatly enhance overall system performance. Note that the term *change* is used, to indicate that the relevant algorithms can be used in cases where a change in a system parameter is not necessarily the result of a fault/failure but a "normal" parameter change.

A three-stage method is proposed which is capable of performing all the necessary tasks of a change monitoring system in the case of additional sensor noise. Using GLR system modeling, the effect of increased sensor noise covariance on the joint pdf of the Kalman filter innovations is calculated. By operating on sliding windows of innovations data, hypothesis testing on the innovations variance is used to decide whether a change has occurred or not. The use of sliding windows of data increases the sensitivity of the detection mechanism. Also, recursive window relations are used for the calculation of the necessary sample statistics, thus making the whole procedure suitable for an on-line computerised diagnostic/supervising system. Following a positive change decision, the estimated innovations variance is used to calculate the new sensor noise covariance. Finally, the relevant Kalman filter and change monitoring parameters are reinitialised to reflect the latest findings.

2. Problem statement

Following GLR fault detection ideas [13], the monitored process is modeled as:

$$x(k+1) = \Phi x(k) + w(k), \tag{1a}$$

$$y(k) = Hx(k) + v(k) + \zeta(k)\sigma_{k,\theta}, \tag{1b}$$

where $x(k) \in \Re^n$ is the state with gaussian initial condition $x(0)$ of mean $\hat{x}_0$ and covariance P_0, $y \in \Re^p$ is the observation sequence and $\{w(k)\}$, $\{v(k)\}$ are independent, zero mean, white Gaussian sequences with $E\{w(k)w(k)^{\mathrm{T}}\} = Q$ and $E\{v(k)v(k)^{\mathrm{T}}\} = R$. Also the noise sequence $\zeta(k)$, modeling the additional sensor noise, is conveniently defined as gaussian of zero mean and unknown constant covariance S independent of $x(0)$, $w(i)$, $v(i)$ for all i, k. Finally $\sigma_{k,\theta}$ is the step function which is unity if $k \geqslant \theta$ and zero otherwise. In this context θ models the unknown change onset time, and is infinite if no change occurs. The system is assumed to be well behaved, i.e., uniformly completely controllable and observable, ensuring stability of its Kalman filter estimator.

If no additional sensor noise is present, the statistical properties of the residuals are:

– Gaussian distribution, whiteness, zero mean and covariance given by,

$$\operatorname{cov}\{\gamma(k),\gamma(m)\} = C_{norm}(k,m) = HP(k/k-1)H^{\mathrm{T}} + R; \quad k = m, \qquad \qquad = 0 \quad ; \quad k \neq m, \tag{2}$$

where $P(.)$ is the state estimate error covariance. Furthermore, if $P(.)$ has settled to its steady value P, the innovations sequence is stationary.

However, if additional sensor noise exists, as shown in [13], the covariance matrix of the innovations can be expressed as the sum of two distinct terms:

$$\begin{aligned}\operatorname{cov}\{\gamma(k),\ \gamma(m)\} &= C_{norm}(k,m) + \sum_{i=\theta}^{\min\{k,m\}} G(k,i)SG^{\mathrm{T}}(m,i) \\ &= C_{norm}(k,m) + C_{fail}(k,m).\end{aligned} \tag{3}$$

This expression shows that the covariance matrix is no longer block diagonal, indicating correlated residuals which, however, retain their zero mean property. Furthermore, the residual sequence is not stationary. Here, the $G(k,\theta)$ are *signature matrices* which can be precomputed using the recurrence relations,

$$F(k,\theta) = G(k,\theta) = 0; \quad k < \theta, \tag{4a}$$

$$G(\theta,\theta) = I, \tag{4b}$$

$$F(k,\theta) = K(k)G(k,\theta) + \Phi F(k-1,\theta); \quad k \geqslant \theta, \tag{4c}$$

$$G(k,\theta) = -H\Phi F(k-1,\theta); \quad k > \theta, \tag{4d}$$

and $K(k)$ is the Kalman gain.

These properties mean that the joint pdf of the innovations can be completely characterised by its first and second moments. It is appropriate in fault detection situations to consider sliding windows of data in order to achieve higher sensitivity and fast detection times. To ease notational complexity, the following definitions are made for an innovations window containing n_d samples at time instant k:

$$\begin{aligned}(\gamma^{k,n_d})^{\mathrm{T}} &= \left[\gamma^{\mathrm{T}}(j)\gamma^{\mathrm{T}}(j+1)\ldots\gamma^{\mathrm{T}}(k)\right], \\ C^{k,n_d} &= \operatorname{cov}\{\gamma^{k,n_d},\gamma^{k,n_d}\}, \\ &= E\{\gamma^{k,n_d}(\gamma^{k,n_d})^{\mathrm{T}}\} \in \Re^{p^*\times p^*},\end{aligned}$$

where $j = k - n_d + 1$ and $p^* = p \times n_d$. Using these definitions the pdf of the windowed Gaussian vector

γ^{k,n_d} is, in general [13],

$$p(\gamma^{k,n_d} \mid \theta, S) = \frac{\exp\left\{ -\frac{1}{2}(\gamma^{k,n_d})^{\mathrm{T}}(C^{k,n_d})^{-1}\gamma^{k,n_d} \right\}}{(2\pi)^{p^*/2}|C^{k,n_d}|^{1/2}}$$

$$= \left(\prod_{m=j}^{\theta-1} \frac{\exp\left\{ -\frac{1}{2}\gamma^{\mathrm{T}}(m)(C_{norm}(m,m))^{-1}\gamma(m) \right\}}{(2\pi)^{p/2}|C_{norm}(m,m)|^{1/2}} \right) \tag{5}$$

$$\times \frac{\exp\left\{ -\frac{1}{2}(\gamma^{k,n_\theta})^{\mathrm{T}}(C^{k,n_\theta})^{-1}\gamma^{k,n_\theta} \right\}}{(2\pi)^{q^*/2}|C^{k,n_\theta}|^{1/2}},$$

where $n_\theta = (k - \theta + 1)$, $q^* = p \times n_\theta$,

$$C^{k,n_d} = \begin{bmatrix} C_{norm}^{\theta-1,n_d-n_\theta} & 0 \\ 0 & C_{norm}^{k,n_\theta} + C_{fail}^{k,n_\theta} \end{bmatrix},$$

$$C_{norm}^{i,j} = \operatorname{diag}\,[C_{norm}(m,m)], \quad m = i - j + 1, \ldots, i,$$

$$C_{fail}^{k,n_\theta} = \begin{bmatrix} C_{fail}(\theta,\theta) & C_{fail}(\theta+1,\theta) & \ldots & C_{fail}(k,\theta) \\ C_{fail}(\theta+1,\theta) & C_{fail}(\theta+1,\theta+1) & \ldots & C_{fail}(k,\theta+1) \\ \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots & \ldots\ldots\ldots \\ C_{fail}(k,\theta) & \ldots\ldots\ldots & \ldots & C_{fail}(k,k) \end{bmatrix}$$

and

C^{k,n_d} is $[p^*] \times [p^*]$,

$C_{norm}^{\theta-1,n_d-n_\theta}$ is $[p \times (\theta - n_d + n_\theta)] \times [p \times (\theta - n_d + n_\theta)]$,

C_{fail}^{k,n_θ} is $[p \times n_\theta] \times [p \times n_\theta]$.

As seen the resulting window covariance matrix consists, in general, of two blocks. The first block is diagonal and corresponds to the no-change period, while the second is no longer diagonal and corresponds to change occurrence. If no change occurs, θ is infinite and all formulae reduce to the standard ones.

3. The CDI procedure

The CDI procedure is based on the fact that an increase in the sensor noise covariance causes a predictable change in the innovations covariance. Furthermore, this is the only effect produced, i.e., the innovations mean remains zero. Equation (3) shows that there are two characteristics that one may test to detect additional sensor noise: innovations variance and innovations whiteness property. Previous analysis, described in [15], has shown that the variance detector is a much faster, reliable and simpler detector than various statistics used to test whiteness (first order serial correlation, Kendall's τ etc., see for example [10]. Therefore this statistic is adopted. The whole procedure consists of three stages: a detection/isolation stage, an estimation stage and a reorganization stage. This results in faster detection rates, since the data window used in the detection stage is smaller than the one used in the estimation stage, where accuracy is important. Next, each stage is presented in detail.

a) Detection. In order to use clasical statistical estimation and hypothesis testing theory, the innovations sequence must be *stationary*, i.e., the filter must *be in steady state.* In this situation,

$$C_{norm}(k,k) = HPH^{\mathrm{T}} + R = C_{norm}. \tag{6}$$

As mentioned earlier, moving windows of innovations data of length n_d are used for the detection phase. Therefore at each time $k > k_d$ (k_d being the settling time of the filter), the following hypothesis tests are performed:

1. Test for zero mean:

$$\mathrm{H_0}:\ E\{\gamma(i)\} = 0 \tag{7}$$

against

$$\mathrm{H_1}:\ E\{\gamma(i)\} \neq 0; \quad i = k - n_d + 1, \ldots, k.$$

Starting time k_d depends on the dynamics of the system, that is on the value of the largest eigenvalue. The statistic adopted here is the component sign test [4], which is a non-parametric test, more robust in this case, since the innovations sequence may or may not be white. This test uses the sign statistic for each component of the vectors and combines them in a quadratic form. If we define,

$$S = (S_1 \ldots S_p),$$

where

$$S_i = \sum_{m=j}^{k} \operatorname{sgn} \gamma_i(m); \quad j = k - n_d + 1,$$

and the sign function defined in the usual way as,

$$\operatorname{sgn}(z) = \begin{cases} 1, & z > 0, \\ 0, & z = 0, \\ -1, & z < 0, \end{cases}$$

then we can form,

$$S^* = S^{\mathrm{T}} \widehat{W}^{-1} S, \tag{8}$$

where

$$\widehat{W}_{i,\ell} = \sum_{m=j}^{k} \operatorname{sgn} \gamma_i(m) \operatorname{sgn} \gamma_\ell(m) \quad \text{for } 1 \leqslant i \leqslant p \text{ and } 1 \leqslant \ell \leqslant p.$$

The test rejects $\mathrm{H_0}$ (detects change) for values of S^* greater than χ^2_p.

2. Test for given covariance matrix, that is,

$$\mathrm{H_0}:\ \operatorname{cov}\{\gamma(i), \gamma(i)\} = C_{norm}$$

against

$$\mathrm{H}_1:\ \mathrm{cov}\{\gamma(i),\gamma(i)\} \neq C_{norm};\quad i = k - n_d + 1,\ldots,k.$$

The commonly used statistic for testing covariance equality, is the sample covariance matrix, calculated at time instant k from a sample of size n_d by,

$$\widehat{C}^{k,n_d} = \frac{1}{n_d - 1}\sum_{i=j}^{k}\gamma(i)\gamma^{\mathrm{T}}(i), \tag{9}$$

where $j = k - n_d + 1$. It should be noted that division by $(n_d - 1)$ in (9) instead of n_d, produces unbiased estimates, and is therefore preferred for small window lengths appropriate for fast detection times. Also the zero mean property has been used.

For change detection, window length and probabilities of false alarm P_f, and correct detection P_d, should be specified and related. Recall that P_f is the significance level of the test or its probability of type-I error, while P_d is the power of the test or the complementary probability of a type-II error. Since the tests are different for the scalar and vector cases, they are considered separately.

I. Scalar case: Since the sample variance is distributed as a χ^2 variable with $n_d - 1$ degrees of freedom, the equations relating P_f, P_d and n_d are the following [10]:

$$\chi^2_{n_d-1;P_f/2} < \frac{(n_d - 1)\hat{c}^{k,n_d}}{c_{norm}} < \chi^2_{n_d-1;1-P_f/2}, \tag{10}$$

$$P_d = P\left[\chi^2_{n_d-1} < \lambda\chi^2_{n_d-1;P_f/2}\right] + P\left[\chi^2_{n_d-1} > \lambda\chi^2_{n_d-1;1-P_f/2}\right], \tag{11}$$

where $\lambda = c_{norm}/\hat{c}^{k,n_d}$ and $\chi^2_{n;\alpha}$ denotes the 100α percentage point of the χ^2 distribution with n degrees of freedom, i.e.,

$$\alpha = P\left[\chi^2 > \chi^2_{n;\alpha}\right].$$

There are two equations but four design parameters and therefore specifying P_f, P_d and λ may not yield an acceptable n_d (too large). In this case a tradeoff between fast detection (small n_d) and high P_d (large n_d) is made. The parameter λ deserves some notice. As expected, P_d is a function of the alternative hypothesis, namely the value of the unknown λ. It measures, in a sense, the amount of degradation that we want to allow before the change monitoring system triggers and should be set bearing in mind reliability specifications for the whole system. Having set the detection parameters, the detection rule becomes:

If,

$$\hat{c}^{k,n_d} > \frac{c_{norm}}{n_d - 1}(\chi^2_{n_d-1;P_f/2}) \quad : \text{change at time } t_{fail} = k - n_d + t_d,$$

$$\text{otherwise,} \quad : \text{process in control,}$$

where t_d is the detection delay time. Even though the change time θ is not explicitly determined, one can safely approximate its value with the change alarm time instant t_{fail}. The detection delay time t_d is the number of samples containing change information that have to be processed for the change alarm to trigger. This will in general be a function of change size but a good approximation will be half the detection window size. Simulation studies may yield more accurate values. If improved reliability is required, the change alarm could be delayed until a number of n_c consequent changes are detected. In this case, however, since sliding windows are used, the probability of a false alarm is *not* $P_f^{n_c}$ but higher.

II. Multivariable case: In this case a scalar statistic obtained from maximum likelihood principles is,

$$W^* = (n_d - 1)\{ -p - \ln(|\widehat{C}^{k,n_d}|) + \ln |C_{norm}| + \mathrm{tr}\,(C_{norm}^{-1}\widehat{C}^{k,n_d})\}.$$

In (Anderson, 1984) it is shown that W^* is asymptotically distributed as χ^2 with $p(p+1)/2$ degrees of freedom. For small samples, the scaled statistic,

$$W' = \left\{1 - \frac{1}{6(n_d-1)}\left(2p+1-\frac{2}{p+1}\right)\right\}W^*$$

is suggested in [2]. Since the distribution of either statistic is χ^2, the selection of design parameters and decision rule are similar to the scalar case.

b) Estimation. Following a positive change decision, the estimation stage is entered. At this stage, use is made of the asymptotic properties of the filter and additional sensor noise effects. In particular, investigation of the steady state value of the diagonal elements of C_{fail}^{k,n_θ}, i.e., when $k \gg \theta$ is necessary. As shown in Appendix 1, $C_{fail}(k,k)$ converges to a steady state value C_{fail}, assuming the system is well behaved. This value satisfies,

$$C_{fail} = H\Phi\Sigma(H\Phi)^{\mathrm{T}} + S, \tag{12}$$

$$\Sigma = (I - KH)\Phi\Sigma[(I-KH)\Phi]^{\mathrm{T}} + KSK^T. \tag{13}$$

These two equations must be solved simultaneously to obtain C_{fail}. To do this, pre- and post multiply (12) by K and K^{T}, respectively, to get,

$$KSK^{\mathrm{T}} = KC_{fail}K^T - KH\Phi\Sigma(KH\Phi)^{\mathrm{T}} \tag{14}$$

and by substituting (14) into (13) gives,

$$\Sigma = (I-KH)\Phi\Sigma[(I-KH)\Phi]^{\mathrm{T}} + KC_{fail}K^{\mathrm{T}} - KH\Phi\Sigma(KH\Phi)^{\mathrm{T}}. \tag{15}$$

Furthermore, by (3)

$$C_{fail} = \mathrm{cov}\,\{\gamma(k),\gamma(k)\} - C_{norm}$$

and by substituting into (12) and (15) yields,

$$S = \mathrm{cov}\,[\gamma(k),\gamma(k)] - C_{norm} - H\Phi\Sigma(H\Phi)^{\mathrm{T}}, \tag{16a}$$

$$\Sigma = (I-KH)\Phi\Sigma[(I-KH)\Phi]^{\mathrm{T}} + K[\mathrm{cov}\,\{\gamma(k),\gamma(k)\} - C_{norm}]K^{\mathrm{T}} - KH\Phi\Sigma(KH\Phi)^{\mathrm{T}}. \tag{16b}$$

In order to solve (16a)–(16b) the residual covariance, cov $\{\gamma(k),\gamma(k)\}$, has to be substituted by its estimate $\widehat{C}^{k,n_e}$. This is calculated once from a sample of size n_e at time instant $k = t_{fail} + n_e$ using (9) as,

$$\widehat{C}^{k,n_e} = \frac{1}{n_e - 1}\sum_{i=j}^{k}\gamma(i)\gamma^{\mathrm{T}}(i),$$

where $j = k - n_e + 1$. Note that for estimation we use a different window, n_e, usually larger than n_d since good estimates are desired. This issue is explained in more detail later. Hence, the system to be solved for S is,

$$S = \widehat{C}^{k,n_e} - C_{norm} - H\Phi\Sigma(H\Phi)^{\mathrm{T}},$$
$$\Sigma = (I - KH)\Phi\Sigma[(I - KH)\Phi]^{\mathrm{T}} + K(\widehat{C}^{k,n_e} - C_{norm})K^{\mathrm{T}} - KH\Phi\Sigma(KH\Phi)^{\mathrm{T}}. \tag{17}$$

This is an algebraic equation in the elements of Σ, which can be solved directly for its distinct $n(n+1)/2$ elements. Let,

$$\sigma = [\sigma_{11}\sigma_{12}\ldots\sigma_{1n}\sigma_{21}\ldots\sigma_{2n}\ldots\sigma_{nn}]$$

be the vector of the unknown elements of Σ. Then, as shown in Appendix 2,

$$\sigma = (I - T)^{-1}l, \tag{18}$$

$$T = \begin{bmatrix} t_{11}^{11} & t_{11}^{12} & \ldots & t_{11}^{nn} \\ t_{12}^{11} & t_{12}^{12} & \ldots & t_{12}^{nn} \\ \ldots & \ldots & \ldots & \ldots \\ t_{nn}^{11} & t_{nn}^{12} & \ldots & t_{nn}^{nn} \end{bmatrix},$$

$$l = [\, l_{11} \quad l_{12} \quad \ldots \quad l_{1n} \quad l_{21} \quad \ldots \quad l_{2n} \quad \ldots \quad l_{nn}\,],$$

$$t_{ij}^{xy} = \varphi_{j,y}\varphi_{i,x} - \varphi_{i,x}\sum_{k=1}^{n} m_{j,k}\varphi_{k,y} - \varphi_{j,y}\sum_{k=1}^{n} m_{i,k}\varphi_{k,x},$$

$$L = K(\widehat{C}^{k,n_e} - C_{norm})K^{\mathrm{T}},$$

$$M = KH$$

(suffices denote respective matrix elements).

Plugging Σ into (17a), produces the desired estimate for S. In particular, if the system is scalar, its solution is,

$$s = (\hat{c}^{k,n_e} - c_{norm})\left(1 - \frac{(n\varphi k)^2}{[(1-kn)\varphi]^2 - 1}\right)^{-1}.$$

In the same manner, it is straightforward to deduce similar results for the cross-correlations of the resisuals. Specifically, it may be proved that they, too, converge to steady state values, and further that the correlation decreases to zero as time lag increases. However, for the purpose of this work, these results are not needed, and are therefore omitted for the sake of simplicity.

As mentioned before, since good estimation of the additional noise covariance depends on the sample innovations covariance calculated using (9), the estimation data window n_e should be chosen large enough for a prespecified accuracy. One should also bear in mind that the window in question should only contain samples which carry information on the change. Consequently, sample count for the window should start after the change alarm. An additional source of error is the fact that following a change detection, the innovations are now correlated resulting in the need for a larger sample than if independence existed. In particular, for scalar processes, it is known that [16],

$$\operatorname{var}\left\{\hat{c}^{k,n_e}\right\} \sim \frac{2(c^{k,n_e})^4}{n_e} \sum_{m=-\infty}^{\infty} \rho^2(m).$$

The usefulness of this expression lies in the fact that even though the true variance c^{k,n_e} and correlations $\rho(m)$ are not known exactly, the accuracy is seen to be $O(1/n_e)$ and directly proportional to the degree of total correlation.

c) System reorganization. Following the estimation stage, two types of corrections to the monitoring system must be made. Firstly, appropriate changes are effected in the Kalman filter equations. The sensor noise covariance matrix R used in (2) must be changed to its newly estimated value given by,

$$R(\text{new}) = R(\text{old}) + \widehat{S}$$

and since the state estimate, which has been calculated using the wrong R, is not optimal, the state error covariance matrix P should be modified to reflect this fact. A possible approach is to set,

$$P(\text{new}) = P_0.$$

Secondly, change decision thresholds appearing in (16), (17) must be updated to reflect the new $\hat{c}^k$. The new thresholds are calculated after the filter, with its updated parameters, has settled again.

4. Implementation issues

The overall CDI procedure is implemented by the following algorithm:

Step 1. *Initialization*: specify P_d, P_f, λ, n_e; determine C_{norm}, K, n_d, k_d off-line.
Step 2. *Detection*: for $k > k_d$ process a window of n_d Kalman filter innovations, and calculate the component sign statistic and the residual sample covariance. If H_0 is true, slide the window and repeat Step 2, otherwise go to Step 3.
Step 3. *Estimation*: Process the next n_e samples of the innovations and calculate the new sensor noise covariance matrix R, while disabling the detection mechanism.
Step 4. *Reorganisation*: Reset R, P. Recalculate C_{norm}, K, k_d. Reset k to 0 and go to Step 1.

As far as on-line applications are concerned, care should be exercised regarding the speed and accuracy of the necessary calculations which are to be performed on-line. In [14] it is shown that for a scalar system, the sample window variance can be calculated recursively using,

$$\hat{\gamma}^k = \hat{\gamma}^{k-1} - \frac{1}{n_d}\delta(k), \tag{19}$$

$$\hat{c}^k = \hat{c}^{k-1} + \frac{1}{n_{d-1}}\left(2\delta(k)\hat{\gamma}^{k-1} - \frac{1}{n_d}\delta^2(k) - \gamma^2(k-n_d) + \gamma^2(k)\right), \tag{20}$$

where

$$\delta(k) = \gamma(k-n_d) - \gamma(k).$$

The same formulae can be applied to the elements of the vector processes.

5. Simulation results

The proposed change monitoring methods were tested on the fifth order system,

$$x(k+1) = \begin{bmatrix} 0.75 & -1.74 & -0.3 & 0 & -0.15 \\ 0.09 & 0.91 & -0.0015 & 0 & -0.008 \\ 0 & 0 & 0.95 & 0 & 0 \\ 0 & 0 & 0 & 0.55 & 0 \\ 0 & 0 & 0 & 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} 24.6 & 0 & 0 & 0 & 0 \\ 0 & 0.83 & 0 & 0 & 0 \\ 0 & 0 & 1.83 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} w(k),$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

The whole simulation was performed using *FAULTLAB*, a fault simulation toolbox described in [17].

On this system additional noise of variance $S = \text{diag}\,[6\ 6]$ was applied at $t = 50$. In Figs 1–4, the results of a single test run are shown.

In Fig. 1, the performance of the whole system in terms of the state estimate error is shown. The state estimate errors for all five states are shown, together with their theoretical and sample 3σ limits. It is seen that before system reorganization, state estimates fall outside their expected limits, while sample limits are much larger than theoretical ones. After system reorganisation (roughly at $t = 390$), the situation is reversed.

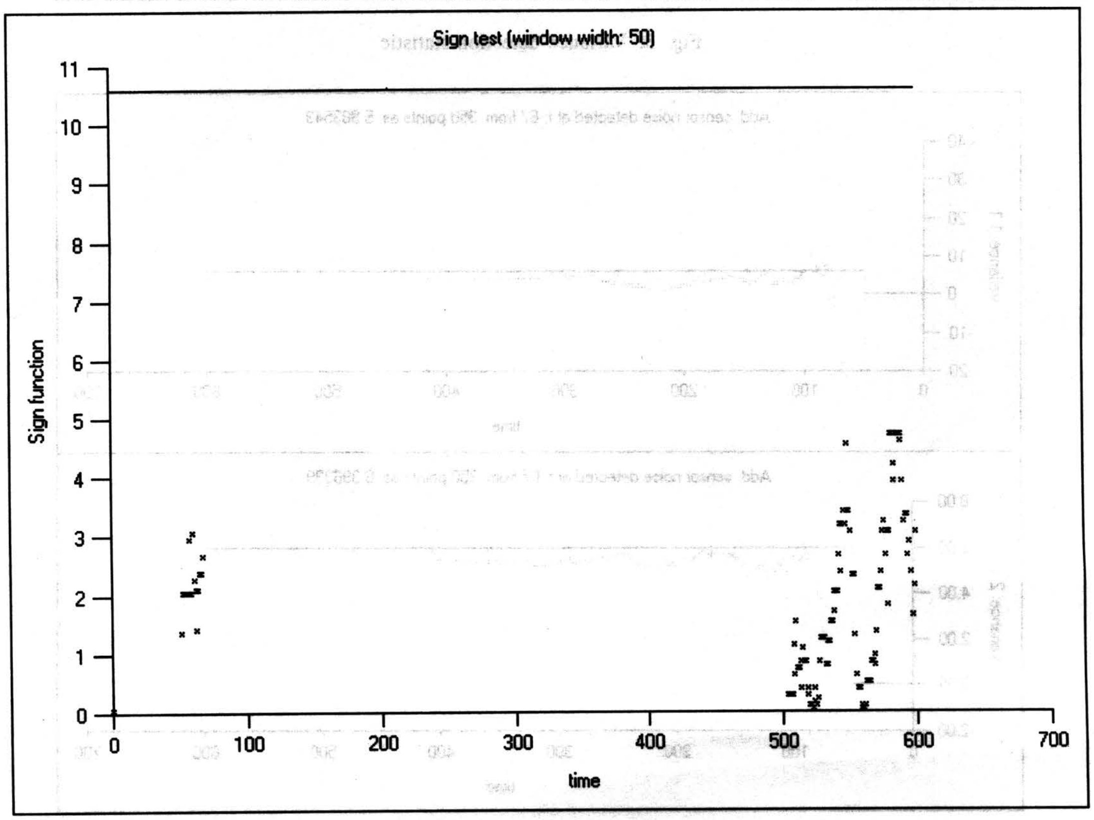

Fig. 2. Behaviour of the sign statistic fault detector

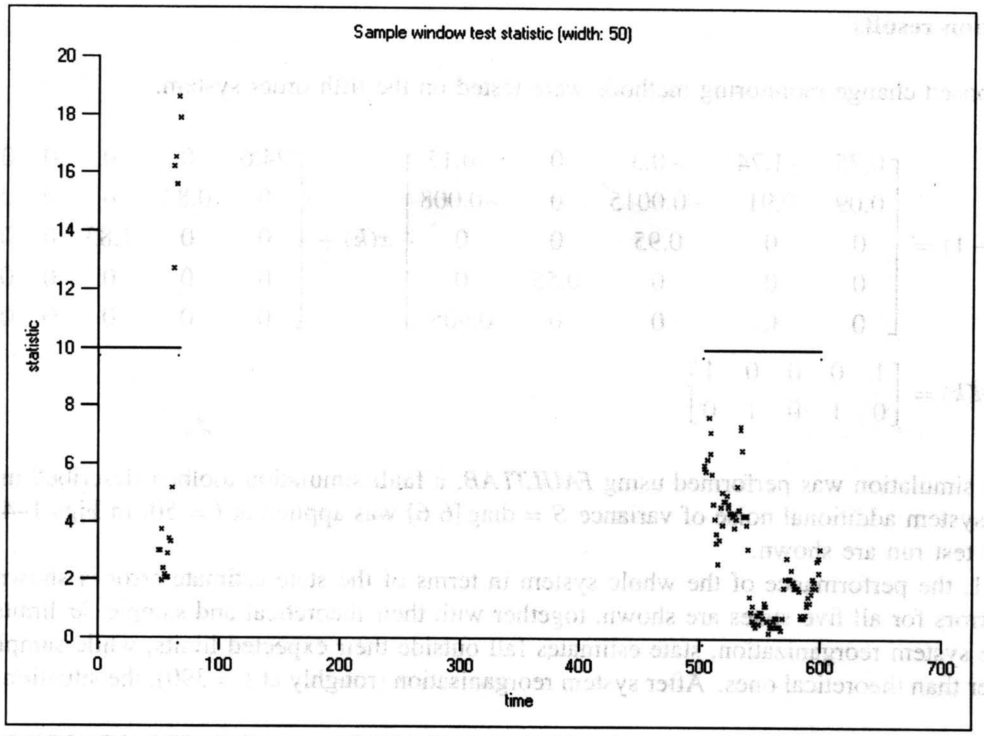

Fig. 3. Variance detection statistic

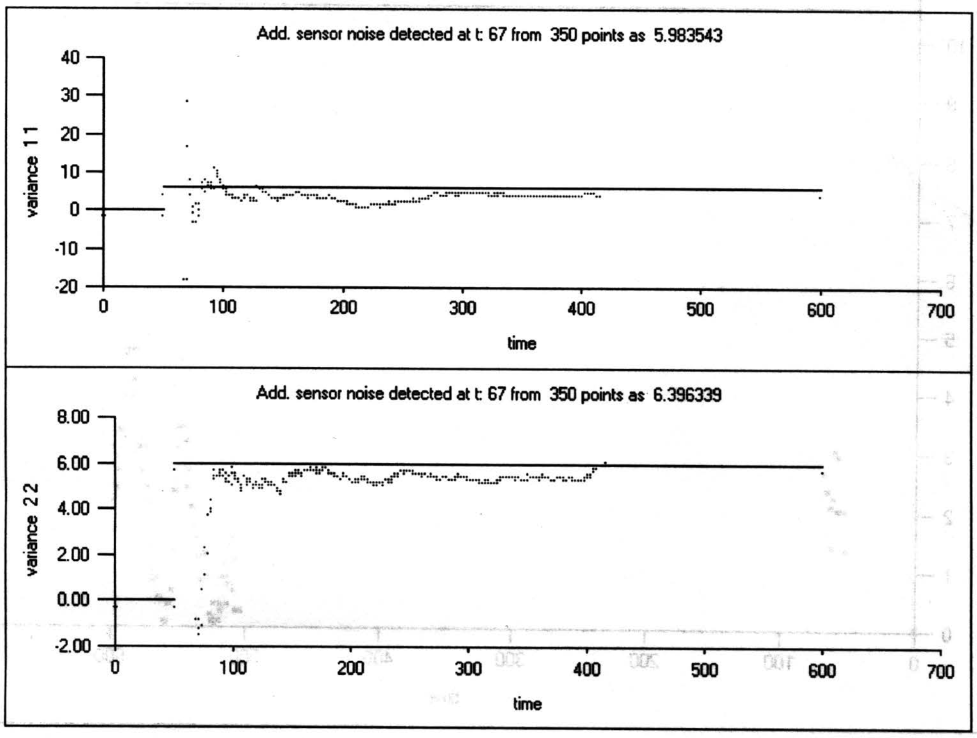

Fig. 4. Estimation of additional sensor noise variance

In Figs 2 and 3 the behavior of the test statistics is shown. Clearly the sign statistic remains within bounds (does not trigger) before and after reorganisation, while the variance test triggers shortly after the fault occurrence ($t = 75$) while it resettles within bounds after system reorganisation. Detection window width was set at $n_d = 50$.

In Fig. 4 the performance of the estimator is shown. Estimation window was set at $n_e = 350$. Both elements of the additional noise covariance are estimated sufficiently clode as 5.98 and 6.4, respectively, thus confirming the applicability of the proposed algorithms.

6. Conclusions

In this paper, an efficient method for the on-line detection of additional sensor noise is presented. In situations where accuracy of incoming information should be maintained at optimum levels, this diagnostic system should prove very valuable. By exploiting the nature of the Kalman filter's innovations behaviour operating in steady state in the event of increased sensor noise and by processing sliding windows of measurements, additional noise is detected, if present, and its variance estimated. Following a change detection, the model is reorganised for optimum performance. All the design parameters can be tuned to reflect desired characteristics. The feasibility of the proposed method is illustrated via a computer simulation of a multivariable system.

Appendix 1

Substituting (4d) into (4c) gives for $k > \theta$,

$$F(k,\theta) = -KH\Phi F(k-1,\theta) + \Phi F(k-1,\theta)$$
$$= (I - KH)\Phi F(k-1,\theta)$$
$$= \prod_{\theta+1}^{k} (I - KH)\Phi F(\theta,\theta)$$

and since by (4c), $F(\theta,\theta) = K$, it follows that,

$$F(k,\theta) = \{(I - KH)\Phi\}^{k-\theta} K; \quad k \geqslant \theta.$$

Hence, using (4d),

$$G(k,\theta) = -H\Phi\{(I - KH)\Phi\}^{k-\theta-1} K; \quad k > \theta, \tag{A1}$$

$$G(\theta,\theta) = I. \tag{4b}$$

Substituting (A1) and (4b) into (3) gives,

$$C_{fail}(k,k) = H\Phi\Big(\sum_{i=\theta}^{k-1} \{(I-KH)\Phi\}^{k-i} KSK^{\mathrm{T}}\{\{(I-KH)\Phi\}^{k-i}\}^{\mathrm{T}}\Big)(H\Phi)^{\mathrm{T}} + S$$

$$= Y\Big(\sum_{i=\theta}^{k-1} W^{k-i} Z (W^{k-i})^{\mathrm{T}}\Big) Y^{\mathrm{T}} + S,$$

where

$$Y = H\Phi, \qquad Z = KSK^{\mathrm{T}}, \qquad W = (I - KH)\Phi.$$

Without loss of generality, let $\theta = 0$. Then,

$$C_{fail}(k,k) = Y\{W^{k-1}Z(W^{k-1})^{\mathrm{T}} + W^{k-2}Z(W^{k-2})^{\mathrm{T}} + \cdots + Z\}Y^{\mathrm{T}} + S.$$

Now, consider the inner matrix sum,

$$\begin{aligned}\Sigma_{k-1} &= W^{k-1}Z(W^{k-1})^{\mathrm{T}} + W^{k-2}Z(W^{k-2})^{\mathrm{T}} + \cdots + Z \\ &= W\{W^{k-2}Z(W^{k-2})^{\mathrm{T}} + W^{k-3}Z(W^{k-3})^{\mathrm{T}} + \cdots + Z\}W^{\mathrm{T}} + Z \\ &= W\Sigma_{k-2}W^{\mathrm{T}} + Z.\end{aligned}$$

This is a matrix difference equation in Σ, which has a finite limit Σ_∞ satisfying,

$$\Sigma_\infty = W\Sigma_\infty W^{\mathrm{T}} + Z$$

since $W = (I - KH)\Phi$ is a stable matrix [12].

Hence, dropping the infinity subscripts, and substituting the appropriate matrices,

$$\begin{aligned}C_{fail} &= H\Phi\Sigma(H\Phi)^{\mathrm{T}} + S, \\ \Sigma &= (I - KH)\Phi\Sigma[(I - KH)\Phi]^{\mathrm{T}} + KSK^{T}.\end{aligned}$$

Appendix 2

The system to be solved for S is,

$$S = \widehat{C}^{k,n_e} - C_{norm} - H\Phi\Sigma(H\Phi)^{\mathrm{T}}, \tag{17a}$$

$$\Sigma = (I - KH)\Phi\Sigma[(I - KH)\Phi]^{\mathrm{T}} + K(\widehat{C}^{k,n_e} - C_{norm})K^{\mathrm{T}} - KH\Phi\Sigma(KH\Phi)^{\mathrm{T}}. \tag{17b}$$

Let us first solve for Σ the second of the above equations. Letting,

$$\begin{aligned}M_1 &= K(\widehat{C}^{k,n_e} - C_{norm})K^{\mathrm{T}}, \\ M_2 &= KH\end{aligned}$$

and performing the operations, (17b) becomes,

$$\Sigma = \Phi\Sigma\Phi^{\mathrm{T}} - \Phi\Sigma\Phi^{\mathrm{T}}M_2^{\mathrm{T}} - M_2\Phi\Sigma\Phi^{\mathrm{T}} + M_1. \tag{A2}$$

Now, examine each term of (A2). For matrix $\Phi\Sigma\Phi^{\mathrm{T}}$ the element (i,j) is, after carrying out the matrix multiplications,

$$(\Phi\Sigma\Phi^{\mathrm{T}})_{i,j} = \sum_{k=1}^{n}\left(\sum_{\ell=1}^{n}\varphi_{i,\ell}\sigma_{\ell,k}\right)\cdot\varphi_{j,k}. \tag{A3}$$

Consequently, the element (i, j) of $(\Phi\Sigma\Phi^{\mathrm{T}})M_2^{\mathrm{T}}$ is,

$$\begin{aligned}((\Phi\Sigma\Phi^{\mathrm{T}})M_2^{\mathrm{T}})_{i,j} &= \sum_{k=1}^{n} (\Phi\Sigma\Phi^{\mathrm{T}})_{i,k} (M_2^{\mathrm{T}})_{k,j} \\ &= \sum_{k=1}^{n}\left(\sum_{m=1}^{n}\left(\sum_{\ell=1}^{n}\varphi_{i,\ell}\sigma_{\ell,m}\right)\cdot\varphi_{k,m}\right)(M_2^{\mathrm{T}})_{k,j}.\end{aligned} \tag{A4}$$

Transposing (A4), yields for $M_2\Phi\Sigma\Phi^{\mathrm{T}}$,

$$(M_2\Phi\Sigma\Phi^{\mathrm{T}})_{i,j} = \sum_{k=1}^{n}(M_2)_{i,k}\left(\sum_{m=1}^{n}\left(\sum_{\ell=1}^{n}\varphi_{k,\ell}\sigma_{\ell,m}\right)\varphi_{j,m}\right). \tag{A5}$$

Hence, the RHS of (A2) becomes,

$$\begin{aligned}&(\Phi\Sigma\Phi^{\mathrm{T}} - \Phi\Sigma\Phi^{\mathrm{T}}M_2^{\mathrm{T}} - M_2\Phi\Sigma\Phi^{\mathrm{T}} + M_1)_{i,j} \\ &= \sum_{m=1}^{n}\left(\sum_{\ell=1}^{n}\varphi_{i,\ell}\sigma_{\ell,m}\right)\varphi_{j,m} - \sum_{k=1}^{n}\sum_{m=1}^{n}\left(\sum_{\ell=1}^{n}\varphi_{i,\ell}\sigma_{\ell,m}\right)\varphi_{k,m}(M_2)_{j,k} - \\ &\quad - \sum_{k=1}^{n}(M_2)_{i,k}\sum_{m=1}^{n}\left(\sum_{\ell=1}^{n}\varphi_{k,\ell}\sigma_{\ell,m}\right)\varphi_{j,m} + (M_1)_{i,j} \\ &= \sum_{m=1}^{n}\varphi_{j,m}\left(\sum_{\ell=1}^{n}\varphi_{i,\ell}\sigma_{\ell,m}\right) - \sum_{k=1}^{n}(M_2)_{j,k}\sum_{m=1}^{n}\varphi_{k,m}\left(\sum_{\ell=1}^{n}\varphi_{i,\ell}\sigma_{\ell,m}\right) - \\ &\quad - \sum_{k=1}^{n}(M_2)_{i,k}\sum_{m=1}^{n}\varphi_{j,m}\left(\sum_{\ell=1}^{n}\varphi_{k,\ell}\sigma_{\ell,m}\right) + (M_1)_{i,j}.\end{aligned} \tag{A6}$$

Equating this with its LHS, produces a set of simultaneous equations for the elements of σ. Thus the next step is to find the coefficients of each $\sigma_{i,j}$ of (A6). Expanding each term in (A6) and collecting terms of same index, produces:

$$\begin{aligned}&(\Phi\Sigma\Phi^{\mathrm{T}} - \Phi\Sigma\Phi^{\mathrm{T}}M_2^{\mathrm{T}} - M_2\Phi\Sigma\Phi^{\mathrm{T}} + M_1)_{i,j} = \sigma_{i,j} \\ &= \sigma_{1,1}\left[\varphi_{j,1}\varphi_{i,1} - \varphi_{i,1}\sum_{k=1}^{n}(M_2)_{j,k}\varphi_{k,1} - \varphi_{j,1}\sum_{k=1}^{n}(M_2)_{i,k}\varphi_{k,1}\right] + \\ &\quad + \sigma_{2,1}\left[\varphi_{j,1}\varphi_{i,2} - \varphi_{i,2}\sum_{k=1}^{n}(M_2)_{j,k}\varphi_{k,1} - \varphi_{j,1}\sum_{k=1}^{n}(M_2)_{i,k}\varphi_{k,2}\right] + \\ &\quad + \cdots + (M_1)_{i,j}.\end{aligned} \tag{A7}$$

Writing (A7) in vector-matrix form produces the desired result:

$$\sigma = T\sigma + m_1 \Rightarrow \sigma = (I - T)^{-1}m_1,$$

where

$$\sigma = [\sigma_{11}\sigma_{12} \ldots \sigma_{1n}\sigma_{21} \ldots \sigma_{nn}],$$

$$m_1 = [(M_1)_{11}\,(M_1)_{12}\,\ldots\,(M_1)_{1n}\,(M_1)_{21}\,\ldots\,(M_1)_{nn}],$$

$$T = \begin{bmatrix} t_{11}^{11} & t_{11}^{12} & \cdots & t_{11}^{nn} \\ t_{12}^{11} & t_{12}^{12} & \cdots & t_{12}^{nn} \\ \cdots & \cdots & \cdots & \cdots \\ t_{nn}^{11} & t_{nn}^{12} & \cdots & t_{nn}^{nn} \end{bmatrix}$$

and

$$t_{ij}^{xy} = \varphi_{j,y}\varphi_{i,x} - \varphi_{i,x}\sum_{k=1}^{n}(M_2)_{j,k}\varphi_{k,y} - \varphi_{j,y}\sum_{k=1}^{n}(M_2)_{i,k}\varphi_{k,x}.$$

References

[1] T.W. Anderson, An Introduction to Multivariable Analysis, 2nd ed. (Wiley, New York, 1984).

[2] M.S. Bartlett, A note on multiplying factors for various chi-squared approximations, J. of the Royal Statistical Society, Series B, vol. 16 (1954) 296–298.

[3] M. Basseville, A. Benveniste, G. Moustakides and A. Rougée, Detection and diagnosis of changes in the eigenstructure of nonstationary multivariable systems, Automatica 23 (1987) 479–489.

[4] P.J. Bickel, On some asymptotic competitors to Hotelling's T^2, The Annals of Mathematical Statistics 36 (1965) 163–173.

[5] R. Billmann and R. Iserman, Leak detection methods for pipelines, Automatica 23 (1987) 381–387.

[6] F.N. Chowdhury and J.L. Aravena, Jumps in observation noise statistics, in: Proc. 32nd CDC, San Antonio, Texas (1993) 271–273.

[7] J.C. Deckert, M.N. Desai, J.J Deyst and A.S. Willsky, F-8 DFBW sensor failure identification using analytic redundancy, IEEE Trans. Aut. Control 22 (1977) 759–803.

[8] M.N. Desai and A. Ray, A fault detection and isolation methodology-theory and application, in: Proc. American Control Conf., San Diego (1984) 262–270.

[9] D.M. Himmelblau, Fault Detection and Diagnosis in Chemical and Petrochemical Processes (Elsevier, Amsterdam, 1978).

[10] M. Kendall and A. Stuart, The Advanced Theory of Statistics (Charles Griffin Ltd., London, 1979).

[11] J.S.H. Liu, Detection, isolation, and identification techniques for noisy degradation in linear discrete time systems, in: Proceedings, 1977 CDC (1977) 1132–1139.

[12] R.F. Ohap and A.R. Stubberud, Adaptive minimum variance estimation in discrete time linear systems, in: C.T. Leondes (Ed.), Control and Dynamic Systems 12 (Academic Press, 1976) 583–624.

[13] A. Pouliezos and G.S. Stavrakakis, Linear state estimation in the presence of sudden system changes: An expert system, in: P. Borne, S. Tzafestas (Eds.), Applied Modelling and Simulation of Technological Systems (North-Holland, 1987) 41–48.

[14] A. Pouliezos and G.S. Stavrakakis, Real-Time Fault Monitoring of Industrial Processes (Kluwer Academic, Dordrecht, 1994).

[15] A. Pouliezos, G.S. Stavrakakis and G. Tselentis, A two stage real-time fault monitoring system, in: S.G Tzafestas (Ed.), Engineering Systems with Intelligence (Kluwer Academic, 1991) 653–660.

[16] M.B. Priestley, Spectral Analysis and Time Series, vol. 1 (Academic Press, 1981).

[17] G. Tselentis, A. Pouliezos and G. Stavrakakis, FaultLab: A simulation toolbox for fault detection strategies, in: Proc. Int. Conference on Fault Diagnostics, Toulouse, France, April 5–7 (1993) 896–905.

[18] K. Uosaki and M. Kawagoe, Backward SPRT failure detection system for detection of innovation variance change, in: Proc., IFAC Identification and System Parameter Estimation, Beijing, PRC (1988) 1153–1157.

[19] B.P. Zeigler, S.D. Chi and F.E. Cellier, Model-based architecture for high autonomy systems, in: S.G Tzafestas (Ed.), Engineering Systems with Intelligence (Kluwer Academic, 1991) 3–22.