Active Control in Smart Beams

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Abstract

In this paper, an active control system is developed for a flexible beam with piezoelectric components. A finite element formulation for modeling the dynamics of the laminated smart beam with bonded piezoelectric sensors/ actuators is used. The control problem is to keep the beam in equilibrium in the event of external wind disturbances and in the presence of model inaccuracies, using the available measurements and control limits. Also of interest is the maximum disturbance the system can handle, given its piezoelectric voltage limits. Classical optimal linear quadratic regulator is used as benchmark. Taking explicitly into account the uncertainty in the system, the theory of robust H_∞ feedback control is used. Solutions to robust stability and robust performance are shown to be very satisfactory through extensive simulations.

Keywords

Smart Beam; Robust Stability; Robust Performance; Active Control

Introduction

In this paper, we consider the case of vibration of smart structures. The stimulus to a structure may originate from external disturbances or excitations that cause structural vibrations. A smart structure would be able to sense the vibration and generate a controlled actuation to itself so the vibration can be minimized. For vibration control purposes, a number of smart materials can be used as actuators and sensors such as piezoelectric, shape memory, electrostrictive and magnetostrictive materials. The emphasis is on using piezoelectric materials because they have good broadband sensing and actuation properties (Halim D. and Moheimani S.O. Reza, 2002).

Various models of beam structures bonded with piezoelectric materials are proposed, followed by classical assumptions, of beam theory and composite materials (Foutsitzi G, Marinova D, Hadjigeorgiou E and Stavroulakis G., 2003). The models differ in the kinetic assumptions and ways of handling the coupling of beams and piezoelectric sensors and actuators for dynamic analyses. A considerable amount of finite element analyses is carried out on structures with piezoelectric actuators to understand their mechanical behaviour (Huang W. S. and Park H. C., 1993). Various control schemes have been implemented in structural control by the use of piezoelectric devices.

Among the commonly used active control schemes are *LQR, PI*, and H_∞. It is known that if the controller is not robust enough, the uncertainties of the system may destroy the efficiency of the controller. The aim of this work is to design a H_∞, robust controller for a beam bonded with piezoelectric sensors and actuators and to investigate the behaviour of the controlled beam. First of all a detailed sheardeformable (Timoshenko) model for a laminated beam structure is developed. A finite element formulation is presented for the model. Cubic and quadratic Hermitian polynomials are used for the transverse and rotational displacements, respectively. The differential equations are based on the Timoshenko beam theory (Friedman and Kosmatka J. .K., 1993). The governing state equation for control design is established. The numerical simulations carried out on the laminated beam shows that the vibration of the system is significantly suppressed within the permitted actuator voltages.

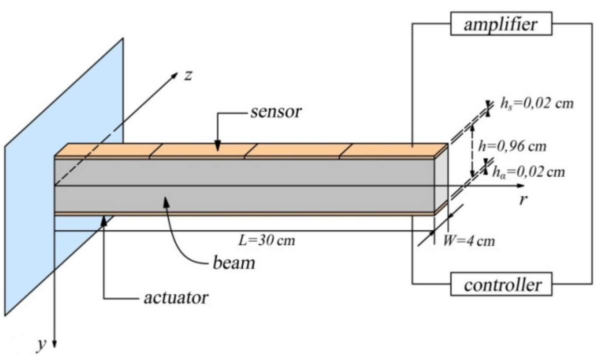

FIG. 1 BEAM WITH PIEZOELECTRIC SENSORS/ACTUATORS

Mathematical Modelling

A cantilever slender beam with rectangular cross-sections is considered. Four pairs of piezoelectric patches are embedded at the top and the bottom surfaces of the beam symmetrically, as shown in Figure 1. The top patches acts like sensors and the bottom ones like actuators. The resulting composite beam is modelled by means of the classical laminated technical theory of bending. Let us assume that the mechanical properties of both the piezoelectric material and the host beam are independent of time. The thermal effects are considered to be negligible.

The equation of motion for a beam element is expressed in terms of nodal variable q as follows (Miara B., Stavroulakis G. and Valente V., 2007, Foutsitzi G., Marinova D., Hadjigeorgiou E. and Stavroulakis G., 2002)

$$M\ddot{q}(t)+D\dot{q}(t)+Kq(t)=f_m(t)+f_e(t) \tag{1}$$

where M is the generalized mass matrix, D the viscous damping matrix, K the generalised stiffness matrix, f_m the external loading vector and f_e the generalised control force vector produced by electromechanical coupling effects. The independent variable q(t) is composed of transversal deflections w_i and rotations ψ_i, i.e.,(Zhang N. and Kirpitchenko I., 2002)

$$q(t)=\begin{bmatrix} w_1 \\ \psi_1 \\ \cdots \\ w_n \\ \psi_n \end{bmatrix} \tag{2}$$

where n is the number of finite elements used in the analysis. Vectors w and f_m are positive upwards. To transform to state-space control representation, let (in the usual manner (Foutsitzi G, Marinova D, Hadjigeorgiou E and Stavroulakis G., 2003)),

$$\dot{x}(t)=\begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \tag{3}$$

Furthermore to express $f_e(t)$ as $Bu(t)$ we write it as $f_e^* u$, where f_e^* is the piezoelectric force for a unit applied on the corresponding actuator, and u represents the voltages on the actuators.

Lastly $d(t)=f_m(t)$ is the disturbance vector.

Then,

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0_{2nx2n} & I_{2nx2n} \\ -M^{-1}K & -M^{-1}D \end{bmatrix} x(t) + \begin{bmatrix} 0_{2nxn} \\ M^{-1} f_e^* \end{bmatrix} u(t) + \begin{bmatrix} 0_{2nx2n} \\ M^{-1} \end{bmatrix} \\ &= Ax(t)+Bu(t)+Gd(t) \\ &= Ax(t)+\begin{bmatrix} B & G \end{bmatrix}\begin{bmatrix} u(t) \\ d(t) \end{bmatrix} \\ &= Ax(t)+\tilde{B}\tilde{u}(t) \end{aligned} \tag{4}$$

We can augment this with the output equation (for example, displacements only measured) (Miara B., Stavroulakis G. and Valente V., 2007),

$$\begin{aligned} y(t) &= \begin{bmatrix} x_1(t) & x_3(t) & \cdots & x_{n-1}(t) \end{bmatrix}^T \\ &= Cx(t) \end{aligned} \tag{5}$$

with,

$$C=\begin{bmatrix} 1 & 0 & \ldots & 0 & 0 & \\ 0 & 0 & 1 & \ldots & & \vdots \\ & & \ddots & & & \vdots \\ 0 & \ldots & 0 & 1 & & 0 \end{bmatrix} \tag{6}$$

In this formulation u is n×1 (at most, but can be smaller), while d is 2n×1. The units used are m, rad, sec and N.

Statement of the Robust Control Problem

The optimal control problem is initially studied for the nominal system, i.e., the beam with known elastic, piezoelectric and viscous properties. A more realistic question concerning the robustness of the control in the presence of defects is also addressed. The fact that the system is influenced by disturbances, such as the wind power, as well as the noise of measurements, is taken into account. The mathematical pattern being used in the design is an approximation of the real one. Further, two control laws for the composite beam are designed in order to suppress the vibrations. Because of its linearity and easy implementation, the linear quadratic regulator (LQR) is presented at first. The response of the controlled nominal and damaged beams is investigated. Taking into account the incompleteness of the information about the eventual damages and external additional influences a robust H_∞ controller is designed. A system analysis is made on condition that the system is not accurate but includes uncertainty that may be related to some kind of damage (Zhang N. and Kirpitchenko I., 2002, Stavroulakis G.E., Foutsitzi G., Hadjigeorgiou E., Marinova D. and Baniotopoulos C.C., 2005)

For practical applications both algorithms need several trial-and-error design iterations in order to provide appropriate control voltages, since the piezoelectric actuators can be depolled by high oscillating voltages. The effectiveness of the proposed control strategies is investigated with the help of numerical simulations.

Control Design

The objective in this section is to determine the optimal vector of active control forces $u(t)$ subjected to performance criteria and to satisfy the dynamical equations of the system, thus reducing in an optimal way the external excitations. We consider the steady state (infinite time) case, i.e. the optimization horizon is allowed to extend to infinity. We seek a linear state feedback (Zhang N. and Kirpitchenko I., 2002)

$$u(t) = -Kx(t) \tag{7}$$

with constant gain K. The control problem is to keep the beam in equilibrium which means zero displacements and rotations in the face of external disturbances, noise and model inaccuracies, using the available measurements (displacement) and controls (Zhang N. and Kirpitchenko I., 2002).

Robustness Analysis

The following three steps are taken in the robustness analysis:

1) Expression of uncertainty set by a mathematical model.

2) Robust stability (RS): check if the system remains stable for all plants within the uncertainty set.

3) Robust performance (RP): if system is robustly stable, check whether performance specifications are met for all plants within the uncertainty set.

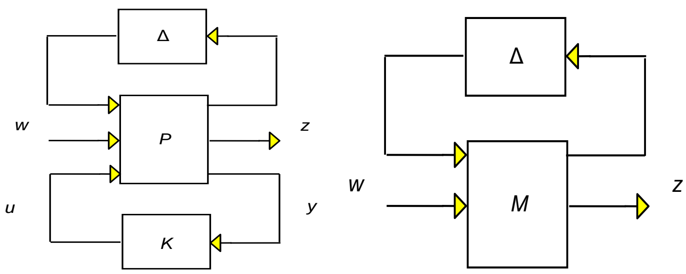

FIG. 2 a, b UNCERTAINTY MODELING

To perform the robustness analysis, the interconnection of Figure 2a will be used. Here P is the nominal plant which includes the uncertainty modelling and K is the calculated H_∞ controller, where M is the nominal system, w is the inputs: noise, disturbance, control and z is the outputs: errors, control extent, measurement. The uncertainty included in Δ which satisfies $\|\Delta\|_\infty \leq 1$. Since K is known, Figure 2a can be simplified to Figure 2b. Given this structure it is known that (Shahian B. and Hassul M., 1994),

I) The system (M,Δ) is robustly stable if,

$$\sup_{w\hat{I}n} \mu_{\Delta}(\mathrm{M}_{11}(jw)) < 1 \tag{8}$$

where,

$$\frac{1}{\mu_{\mathrm{B}}(\mathrm{M})} = \inf_{\Delta \hat{I} \mathrm{B}_{\Delta}, \det(\mathrm{I}-M\Delta)=0} \bar{\sigma}(\Delta) \tag{9}$$

is the structure singular value of M given the structured uncertainty set as B_{Δ}.

II) The system (M,Δ) exhibits robust performance if,

$$\sup_{w\hat{I}n} \mu_{\Delta\alpha}(\mathrm{M}(jw)) < 1 \tag{10}$$

where,

$$\Delta_{\alpha} = \begin{bmatrix} \Delta_{\rho} & 0 \\ 0 & \Delta \end{bmatrix} \tag{11}$$

and Δ_{ρ} has the same structure as Δ but dimensions corresponding to (w,z). Unfortunately, only bounds on μ can be estimated.

To proceed let us assume uncertainty in the M, D and K matrices of the form,

$$\begin{aligned} M &= M_0(I + m_p \delta_{\mathrm{M}u}) D = D_0(I + d_p \delta_D) \\ K &= K_0(I + k_p \delta_{\mathrm{K}}) \end{aligned} \tag{12}$$

with,

$$\|\Delta\|_{\infty} \overset{def}{=} \left\| \begin{bmatrix} \delta_{\mathrm{M}} & & \\ & \delta_D & \\ & & \delta_{\mathrm{K}} \end{bmatrix} \right\| < 1 \tag{13}$$

This means that a percentage deviation from the nominal values is allowed (Sisemore C., Smaili A. and Houghton R., 1999). With these definitions Eq.(13) becomes,

$$\begin{aligned} &\Rightarrow \mathrm{M}_0 \ddot{q}(t) + D_0 \dot{q}(t) + K_0 q(t) \\ &= \tilde{D} q_u(t) + f_m(t) + f_e(t) \end{aligned} \tag{14}$$

where,

$$\begin{aligned} q_u(t) &\overset{def}{=} \begin{bmatrix} \ddot{q}(t) \\ \dot{q}(t) \\ q(t) \end{bmatrix}, \\ \tilde{D} &= -\begin{bmatrix} M_0 m_p & D_0 d_p & K_0 k_p \end{bmatrix} \\ &= \begin{bmatrix} I_{2nx2n}\delta_{\mathrm{M}} & & \\ & I_{2nx2n}\delta_{\Delta} & \\ & & I_{2nx2n}\delta_{\mathrm{K}} \end{bmatrix} \end{aligned} \tag{15}$$

Writing (14) in state space form, gives,

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0_{2n\times 2n} & I_{2n\times 2n} \\ -M^{-1}K & -M^{-1}D \end{bmatrix} x(t) + \begin{bmatrix} 0_{2n\times n} \\ M^{-1} f_e^* \end{bmatrix} u(t) \\ &+ \begin{bmatrix} 0_{2n\times 2n} \\ M^{-1} \end{bmatrix} d(t) + \begin{bmatrix} 0_{2n\times 6n} \\ M^{-1}\tilde{D} \end{bmatrix} q_u(t) \\ \dot{x}(t) &= Ax(t) + Bu(t) + Gd(t) + G_u q_u(t) \end{aligned} \tag{16}$$

In this way uncertainty in the original matrices is treated as an extra uncertainty term.

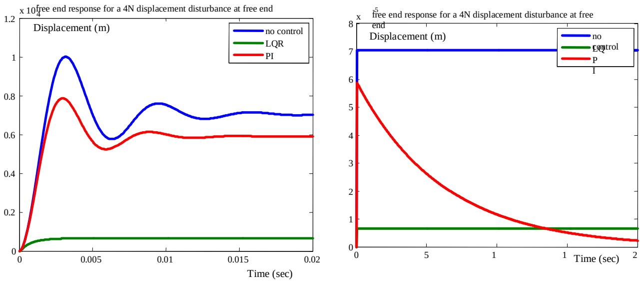

FIG. 3 RESPONSE OF THE FREE VIBRATING BEAM WITH AND WITHOUT CONTROL

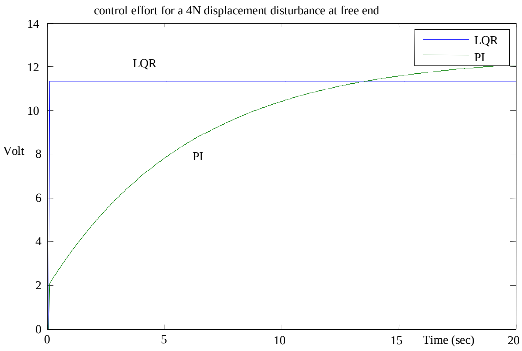

FIG. 4 CONTROL VOLTAGES

Results

For the numerical simulations a cantilevered composite beam with viscous damping and piezoelectric layers bonded on its top and bottom and discretized with four finite elements, is used. A finer finite element discretization, which certainly is required for the approximation of higher frequencies, does not change the trend of the results. The parameters of the beam are similar to those used by Sisemore C., Smaili A. and Houghton R., 1999. Our aim is to study the response of the composite beam in the presence of defects and damages.

Two kinds of dynamic loading are used as disturbances:

a) Transient force 4N distributed in the free end of the beam.

b) Periodic sinusoidal loading pressure acting on the side of the structure simulating a strong wind (Stavroulakis G.E., Foutsitzi G., Hadjigeorgiou E., Marinova D. and Baniotopoulos C.C., 2005). A sinusoidal load with on amplitude of 10N and frequency of 6 32rad/sec has been considered.

Let us first investigate the response of the free and LQR-controlled composite beam with piezoelectric and viscous layers for various parameters of the glue layer. A vertical impulsive load is applied at the free-end of the beam. Figure 3 shows the response of the beams free end to a constant external force of 4N applied to the free end. The allowable voltage of the piezoelectric actuators used for the beam ranges from - 500V to +500V. The control voltage must not exceed this range; otherwise, the actuators will lose their piezoelectricity and fail to work at all. Hence, in the control design process the balance between the vibration control level and control input is considered in order for the piezoelectric actuators to endure the limited input voltage. The control effort is shown in Figure 4, where it is seen that both controllers use comparable voltage and are well within the 500V limit of piezoelectric actuators. With finer tuning (perhaps by adding a D term also), the speed of response and other transient characteristics (overshoot) can of course be improved. All the above assume (and have been produced with) a full state measurement, which is unrealistic.

In Figures 5 - 6 the same experiments are shown using a reduced order (Luenberger) observer (Shahian B. and Hassul M., 1994). In fact the plant can be controlled with just one pair of sensor/actuator piezo Figure 7. Figure 8 shows the response of the uncontrolled beam and controlled beam using H_∞ control strategy. The nominal performance is depicted in Figures 8- 10. The beam with H_∞ control keeps in equilibrium and we have almost zeros displacement. Figure 9 shows the control voltages for the four nodes of the beam. As seen, nominal performance is very satisfactory with controls within limits. Robust performance is shown in Figure 11. The both robuststability and robust performance are less than one (8,9), indicating that our system is robustly stable and exhibits robust performance.

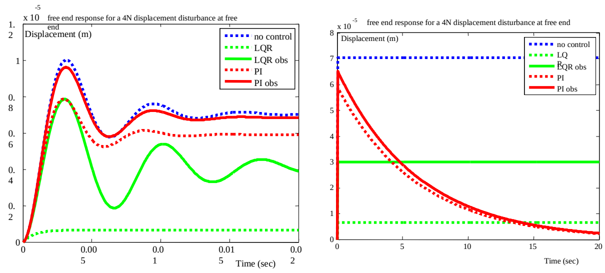

FIG. 5 RESPONSE OF FREE VIBRATING BEAM WITH AND WITHOUT CONTROL USING REDUCE ORDER

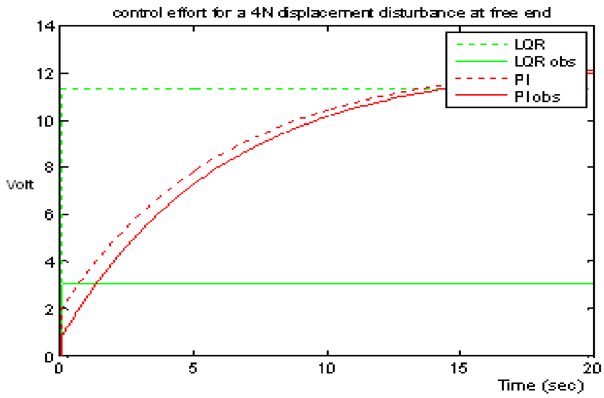

FIG. 6 CONTROL VOLTAGES

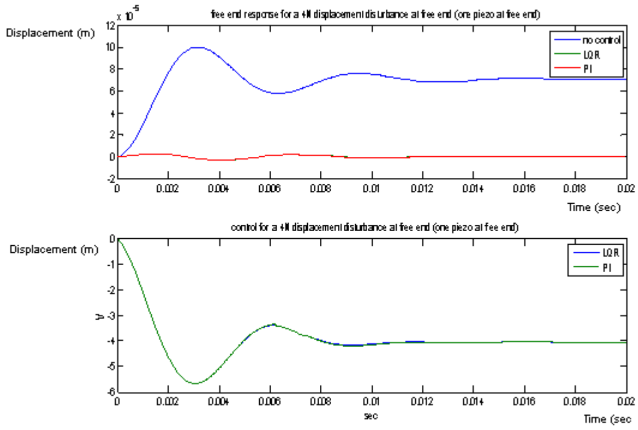

FIG. 7 RESPONSE OF FREE VIBRATING BEAM WITH ONE PAIR OF ACTUATOR

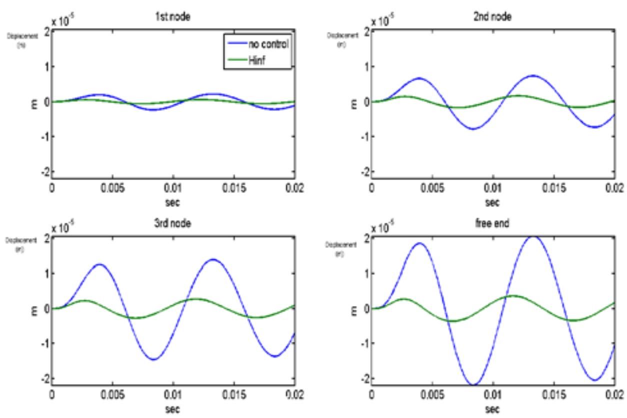

FIG. 8 RESPONSE OF THE FOUR NODES VIBRATING BEAM WITH AND WITHOUT CONTROL

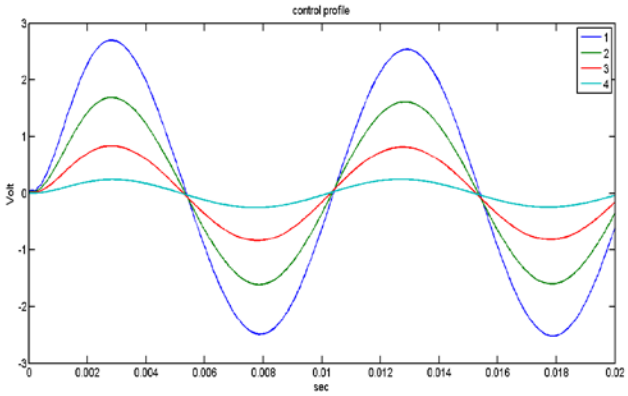

FIG. 9 CONTROL PROFILE FOR THE FOUR NODES

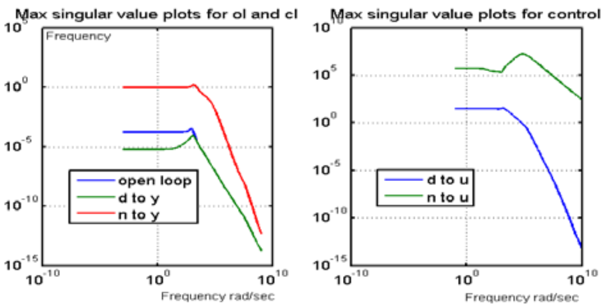

FIG. 10 SINGULAR VALUE

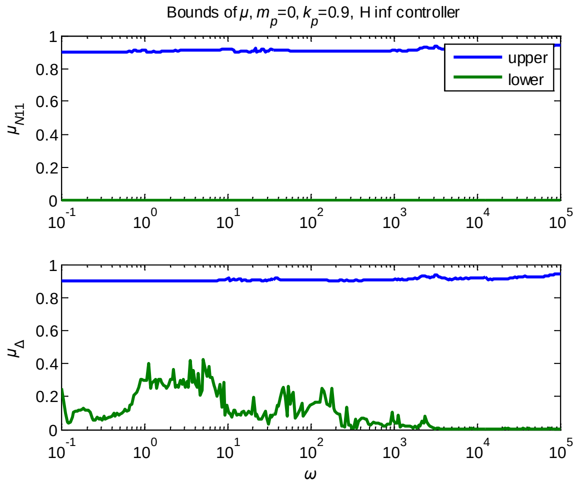

FIG. 11 ROBUST PERFORMANCE

Conclusions

An H_∞ controller was designed which effectively suppressed the vibrations of the beam. The suitability of the H_∞ design technique in the modeling of uncertainties and the evaluating of the robust performance of the system was demonstrated.

After the analysis of the system, we checked the robust stability and system performance. The introduction of uncertainty permited us to keep the structure in service up to given limits of uncertainty. The results showed that the proposed model and method are effective and the control behaviour of the beam achieves the predicted characteristics. Numerical simulations verified the effectiveness and the good quality of the proposed model and the control strategies.

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Anastasios Pouliezos was born in Athens. He received the BSc. Degree in Mathematics and Computing from the Polytechnic of North London in 1975, MSc. in Control Systems from Imperial College, University of London in 1976 and Phd. degree from Brunel University London in 1980.

He is full professor of Control Systems at the Department of Production and Management Engineering of the Technical University of Crete, Greece. Since 1985, he has been a member of the academic staff of the Technical University of Crete. He is the Director of the Industrial Systems Control Laboratory of the Technical University of Crete, which is built around a team of experts and specializes, amongst others, in the areas of automatic control, system fault diagnosis, on-line quality control, on-line health condition monitoring, e-learning, intelligent energy systems, intelligent building systems (BEMS), renewable energy sources, transportation and public utility networks, dynamic optimization algorithms. Dr. Pouliezos published over 100 papers in international journals and conference proceedings in the fields of intelligent control, fuzzy and neural systems for prediction and control, decision support systems, on-line quality control, on-line health condition monitoring, e-learning and intelligent building systems (BEMS).