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A TWO STAGE REAL TIME FAULT MONITORING SYSTEM

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ABSTRACT

In the present paper, a two-stage fault monitoring scheme is proposed and evaluated for discrete linear stochastic systems in the event of sensor noise degradation. In the first stage, fault detection and partial isolation is achieved using simple statistical tests based on the failure effect on the joint pdf of the Kalman-Bucy filter innovations. Complete fault identification (fault size, fault location, time of occurrence) is performed by the subsequent use of generalized likelihood ratio test (GLR). The analysis is verified by computer simulation of a first order discrete stochastic system.

1. INTRODUCTION

Consider the following discrete-time dynamical system:

$$\mathbf{x}(k+1) = \mathbf{\Phi}\,\mathbf{x}(k) + \mathbf{w}(k) \tag{1}$$
$$\mathbf{y}(k) = \mathbf{H}\,\mathbf{x}(k) + \mathbf{v}(k) + \boldsymbol{\zeta}_y(k)\sigma_{k,\theta} \tag{2}$$

where $\mathbf{x}(k) \in \mathbf{R}^n$ is the state with gaussian initial condition $\mathbf{x}(0)$ of mean $\mathbf{x}_0$ and covariance $\mathbf{P}_0$, $\mathbf{y} \in \mathbf{R}^p$ is the observation sequence and $\{\mathbf{w}(k)\}$, $\{\mathbf{v}(k)\}$ are independent, zero mean, white Gaussian sequences with $E[\mathbf{w}(k)\mathbf{w}(k)^T] = \mathbf{Q}$ and $E[\mathbf{v}(k)\mathbf{v}(k)^T] = \mathbf{R}$. Also the noise sequence $\boldsymbol{\zeta}_y(k)$, modelling the additional sensor noise, is conveniently defined as gaussian of zero mean and unknown constant variance $\mathbf{S}_y$ independent of $\mathbf{x}(0)$, $\mathbf{w}(i)$, $\mathbf{v}(i)$ for all i,k. Finally $\sigma_{k,\theta}$ is the step function which is unity if $k \geq \theta$ and zero otherwise. In this context θ models the unknown fault onset time. It should be noted that the time invariance of the system under consideration is used only for simplicity and that extension to time-varying systems is straightforward.

In normal operation, the statistical properties of the residuals are:

S. G. Tzafestas (ed.), Engineering Systems with Intelligence, 653–660.

- Gaussian distribution, whiteness, stationarity, zero mean and covariance given by,

$$\mathbf{C}(k,m) = \mathbf{0},\ k\ m$$
$$\mathbf{C}(k,k) = \mathbf{H}\ \mathbf{P}(k/k-1)\ \mathbf{H}^T + \mathbf{R}$$

where $\mathbf{P}(.)$ is the state estimate error covariance [1].

These properties mean that the joint pdf of the innovations will be completely characterised by its first and second moments. It is appropriate in fault detection situations to consider sliding windows of data in order to achieve fast detection times. To ease notational complexity the following definitions are made for a window of size n_w:

$$(\boldsymbol{\gamma}^{j,k})^T = [\boldsymbol{\gamma}(j)^T\ \boldsymbol{\gamma}(j+1)^T\ \ldots\ \boldsymbol{\gamma}(k)^T] \in R^{p^*},$$
$$\bar{\boldsymbol{\gamma}}^{j,k} = E[\boldsymbol{\gamma}^{j,k}],$$

where $p^* = p \times (k-j+1) = p \times n_w$ and

$$\mathbf{C}^{j,k} = \mathrm{cov}(\boldsymbol{\gamma}^{j,k}) = E[\boldsymbol{\gamma}^{j,k} - \bar{\boldsymbol{\gamma}}^{j,k})\ (\boldsymbol{\gamma}^{j,k} - \bar{\boldsymbol{\gamma}}^{j,k})^T] \in R^{p^* x p^*}$$

Using these definitions the pdf of the gaussian vector $\boldsymbol{\gamma}^{j,k}$ is:

$$p(\boldsymbol{\gamma}^{j,k}) = \frac{1}{(2\pi)^{p^*/2}|\mathbf{C}^{j,k}|^{1/2}}\exp\{-1/2(\boldsymbol{\gamma}^{j,k}-\bar{\boldsymbol{\gamma}}^{j,k})^T(\mathbf{C}^{j,k})^{-1}(\boldsymbol{\gamma}^{j,k}-\bar{\boldsymbol{\gamma}}^{j,k})\}$$

If no fault occurs,

$$p(\boldsymbol{\gamma}^{j,k}) = \prod_{m=j}^{k} \frac{1}{(2\pi)^{p/2}|\mathbf{C}(m,m)|^{1/2}}\exp\{-1/2(\boldsymbol{\gamma}_0(m)^T\mathbf{C}(m,m)^{-1}\boldsymbol{\gamma}_0(m)\} \qquad (3)$$
$$= \pi(j,k)$$

since $E\{\boldsymbol{\gamma}_0(m)\} = \mathbf{0}$ and $\mathbf{C}^{j,k} = \mathrm{diag}[\mathbf{C}(m,m)]$, $m = j, j+1, \ldots, k$. Here $\boldsymbol{\gamma}_0(m)$ denotes the normal operation residual sequence.

2. JOINT PDF OF RESIDUALS IN THE EVENT OF ADDITIONAL SENSOR NOISE

If additional sensor noise exists, it is shown in [4] that the residual covariance matrix is no longer block diagonal, resulting in correlated residuals which retain their zero mean property. The covariance is given by:

$$\mathrm{cov}[\boldsymbol{\gamma}(k)\boldsymbol{\gamma}(m)] = \mathbf{C}(k,m) + \sum_{i=\theta}^{\lambda} \mathbf{G}_f(k,i)\ \mathbf{S}_y\ \mathbf{G}_f^T(m,i), \quad \lambda=\min\{k,m\} \qquad (4)$$
$$= \mathbf{C}_f(k,m)$$

where $\mathbf{G}_f(i,j)$ are *signature matrices* which depend on the specific nature

$$
\begin{aligned}
\mathbf{G}_f(k,\theta) &= -\,\mathbf{H}\,\mathbf{\Phi}\,\mathbf{F}_f(k-1,\theta) && k>\theta \\
\mathbf{F}_f(k,\theta) &= -\,\mathbf{K}(k)\,\mathbf{G}_f(k,\theta) + \mathbf{\Phi}\,\mathbf{F}_f(k-1,\theta) && k\geq\theta \\
\mathbf{G}_f(\theta,\theta) &= \mathbf{I} \\
\mathbf{F}_f(k,\theta) &= \mathbf{G}_f(k,\theta) = \mathbf{0} && k<\theta
\end{aligned}
\tag{5}
$$

The joint pdf of the residual sequence then becomes,

$$
P(\boldsymbol{\gamma}^{j,k}|H_1,\theta,\mathbf{S}_y) = \frac{1}{(2\pi)^{p^*/2}\,|\mathbf{C}_f^{j,k}|^{1/2}} \exp\{-1/2\ [\boldsymbol{\gamma}^{j,k}]^T[\mathbf{C}_f{}^{j,k}]^{-1}[\boldsymbol{\gamma}^{j,k}])\}
$$

where,

$$
\mathbf{C}_f{}^{j,k} = \begin{bmatrix} \mathbf{C}^{j,\theta-1} & & \mathbf{0} \\ & \cdot & \\ \mathbf{0} & \cdot & \mathbf{C}_f^{\theta,k} \end{bmatrix}
$$

and

$$
\mathbf{C}_f{}^{\theta,k} = \begin{bmatrix}
[\mathbf{C}(\theta,\theta)+\mathbf{C}_f(\theta,\theta)] & \mathbf{C}_f(\theta+1,\theta) & \cdots & \mathbf{C}_f(k,\theta) \\
\cdot & \mathbf{C}(\theta+1,\theta+1)+\mathbf{C}_f(\theta+1,\theta+1) & \cdots & \mathbf{C}_f(k,\theta+1) \\
\cdot & \cdot & & \cdot \\
\cdot & \cdot & & \mathbf{C}(k,k)+\mathbf{C}_f(k,k)
\end{bmatrix}
$$

$\mathbf{C}_f{}^{j,k}$ is $[p(k-j+1)]\times[p(k-j+1)]$ $\qquad$ $\mathbf{C}^{j,\theta-1}$ is $[(\theta-j)p]\times[(\theta-j)p]$
$\mathbf{C}_f{}^{\theta,k}$ is $[p(k-\theta+1)]\times[p(k-\theta+1)]$

It should be noted that as shown in [3], the same qualitative effects on the residual sequence are obtained if additional plant noise is present. Therefore a full monitoring scheme should also have the capability to infer about this case as well. This is the topic of current research.

3. TESTING FOR ADDITIONAL MEASUREMENT NOISE

The results obtained in Section 2 for the joint PDF of residuals in the event of additional measurement noise, lead quite naturally to a hypothesis testing formulation of the fault monitoring process. The knowledge of the effects of the fault on the Kalman filter innovations can be used to design a scheme that operates on two stages. The first stage is a simple fault detection mechanism where the hypothesis that the generated residuals belong to the class C_0: {no fault} against the hypothesis that they belong to an alternative faulty class C_1: {zero mean correlated residuals}, will be tested. This mechanism performs partial isolation of the failed parameter (since additional plant noise may also be present). On the sounding of an alarm from this first stage, the second stage mechanism is activated. This mechanism performs the functions of fault isolation, estimation of time of occurrence and size of fault and subsequent system reorganization.

In all the subsequent sections of the paper the scalar output case is considered in order to evaluate the proposed scheme.

a. Testing for the mean.

The test statistic commonly used for testing,

$$H_0: \bar{y}(k) = 0 \quad \text{against}$$
$$H_1: \bar{y}(k) = y_1(k) \neq 0, \quad k = i, \ldots, j; \ j>i.$$

is the sample mean. An important disadvantage of this test however is that it is not robust in the case of correlated measurements [2]. The **sign test** is therefore proposed, which is a non-parametric test used to test hypotheses on the value of the median of a population [2]. Since the residuals are normal under all hypotheses, the median is equal to the mean and therefore this test can be applied to test for zero mean. The sign test procedure is as follows: the number of positive residuals in a window is calculated and compared to two thresholds which depend on the window size n_w and probability of false detection P_f. Thus if,

$$n_1 < \text{number of positive residuals} < n_2: \text{accept } H_0$$
$$\text{otherwise: reject } H_0$$

The percentage points of the symmetric binomial distribution for different n_w and P_f can be used for the sign test as follows:

i. count the number of values above and below zero, say n^+ and n^-.
ii. choose the smallest of the two values, say n^+.
iii. compare n^+ with the entry of the table of symmetric binomial distribution percentage points for chosen n_w and P_f, say n_a.
iv. if $n^+ < n_a$, reject H_0; otherwise accept it.

The sign test is much more robust in departures from independence than the corresponding sample mean test [3].

b. Testing for whiteness.

Testing for whiteness is a common requirement for a number of identification algorithms that appear in the control literature. Methods requiring a large sample size (>500) as the plotting of the sample autocorrelation coefficients, hypothesis testing on the diagonal form of the correlation matrix, Stoica's test and others are inapplicable to the case of on-line fault detection. Three tests for whiteness were investigated in [3] including two parametric and one nonparametric, i.e.:

i. first order serial correlation test
ii. sample variance $\hat{\sigma}^2$ test
iii. rank correlation test.

The rank correlation test did not perform at all well in two tested cases of additional plant and measurement noise, thus it is eliminated from the possible tests of independence even if the reason for the failure is not clear. The sample variance test requires a big amount of numerical calculations, thus it is not recommended for on-line implementations needed in the fault monitoring scheme. The first order serial correlation test of independence is finally adopted here for its simplicity and robustness. Also its null distribution is available for small window sizes [2]. The first order serial correlation of a filter residual window is defined by:

$$r_1 = \frac{n}{n-1} \frac{\sum_{m=i}^{j-1} \{\gamma(m) - \hat{\gamma}^{i,j})(\gamma(m+1) - \hat{\gamma}^{i,j})\}}{\sum_{m=i}^{j} (\gamma(m) - \hat{\gamma}^{i,j})^2} \tag{6}$$

where $\hat{\gamma}$ is the sample mean.

For small sample sizes (<20) more accurate forms may be used. Under the null hypothesis of whiteness the random variable r_1 is distributed asymptotically normal with mean $E(r_1) = -1/(n-1)$ and variance $var(r_1) = (n-2)^2/(n-1)^3$. Sampling experiments on serial correlation distributions suggest that in the null case normal theory remains approximately valid even for n_w=10 or 20, [2]. Confidence limits for hypothesis testing can be found using normal distribution theory. The probabilities P_f and P_d are respectively given by:

$$P_f = P[\ |\rho_1| > z_{.5P_f}]$$

$$P_d = 1 - \{\Phi[-\rho_1 + z_{.5P_f}] - \Phi[-\rho_1 - z_{.5P_f}]\}$$

where z_a is defined by $P[Z>z_a] = \frac{1}{\sqrt{2\pi}} \int_{z_a}^{\infty} e^{-.5z^2} dz = a$.
and ρ_1 is the standardised normal $(r_1 - E(r_1))/var(r_1)^{1/2}$.

γ. GLR tests

In this case the two hypotheses may be written,

$$H_0 : \mathbf{\gamma}(k) = \mathbf{\gamma}_0(k)$$
$$H_1 : \mathbf{\gamma}(k) = \mathbf{\gamma}_0(k) + \mathbf{g}_f(k, \boldsymbol{\xi})$$

where $\boldsymbol{\xi}$ is the vector of unknown fault parameters (k, s_y). Since only H_1 contains $\boldsymbol{\xi}$, the GLR test statistic is [5],

$$\Lambda_g = \frac{p(\mathbf{\gamma}^{i,j} \mid H_1 \; ; \hat{\boldsymbol{\xi}})}{p(\mathbf{\gamma}^{i,j} \mid H_0 \;)} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{7}$$

To evaluate Λ_g one has to maximise the alternative hypothesis function,

$$f(\boldsymbol{\xi}) = p(\mathbf{\gamma}^{i,j} \mid H_1 \; ; \boldsymbol{\xi}) \tag{8}$$

Using (3) and taking logarithms,

$$-2\ln f(\boldsymbol{\xi}) = -2\ln\pi(j,\theta-1) + \ln|\mathbf{C}_f^{\theta,k}| + (\mathbf{\gamma}^{\theta,k})^T(\mathbf{C}_f^{\theta,k})^{-1}\mathbf{\gamma}^{\theta,k} + p(k-\theta+1)\ln 2\pi$$

To maximise $\ln f(\boldsymbol{\xi})$, a double maximisation has to be performed. One way to do this is to fix $\theta=\bar{\theta}$ and minimise,

$$-2\ln f(\bar{\theta},s_y) = -2\ln\pi(j,\bar{\theta}-1)+\ln|\mathbf{C}_f^{\bar{\theta},k}|+(\mathbf{\gamma}^{\bar{\theta},k})^T(\mathbf{C}_f^{\bar{\theta},k})^{-1}\mathbf{\gamma}^{\bar{\theta},k}+p(k-\bar{\theta}+1)\ln 2\pi$$

for every $\bar{\theta}$ in the window. Equivalently we can minimise,

$$f_1(\bar{\theta},s_y) = \ln|\mathbf{C}_f^{\bar{\theta},k}| + (\mathbf{\gamma}^{\bar{\theta},k})^T \; [\mathbf{C}_f^{\bar{\theta},k}]^{-1} \; \bar{\mathbf{\gamma}}^{\theta,k}) \tag{9}$$

since the remaining terms are constant. Let,

$$f(\hat{\theta},\hat{s}_y) = \max_{\theta,s_y} f(\theta,s_y)$$

Then, since

$$-2\ln\pi(j,k) = \sum_{m=j}^{k} \{p\ln 2\pi + \ln(|c(m,m)|) + \gamma^2(m)c^{-1}(m,m)\}$$

$$= (k-j+1)p\ln 2\pi + \sum_{m=j}^{k} \{\ln(|c(m,m)|) + \gamma^2(m)c^{-1}(m,m)\}$$

and

$$-2\ln\pi(j,\bar{\theta}-1) = (\bar{\theta}-j)p\ln 2\pi + \sum_{m=j}^{\bar{\theta}-1} \{\ln(|c(m,m)|) +\gamma^2(m)c^{-1}(m,m)\}$$

we get the modified test statistic,

$$2\ln\Lambda g = 2\ln p(\mathbf{Y}^{i,j} \mid H_1;\xi) - 2\ln p(\mathbf{Y}^{i,j} \mid H_0) \underset{H_0}{\overset{H_1}{\gtrless}} 2\ln \lambda \qquad (10)$$

4. SIMULATION - CONCLUSIONS

The proposed fault monitoring methods were tested on the simulated stable first order system,

$$x(k+1) = 0.7x(k) + w(k)$$
$$y(k) = x(k) + v(k)$$

with $E\{w(k)\} = E\{v(k)\} = 0$, $E\{w^2(k)\} = E\{v^2(k)\} = 0.3$, $x(0)=5$, $x_0=0$, $P_0=0.5$. Additional sensor noise of variance $s_y=1.9$ was introduced at $\theta=35$. To illustrate the effect of additional sensor noise going undetected, a sample run was executed which shows an increase in the state estimate error variance (filter suboptimality). As shown in Fig.1 the sample error variance settles at 0.64, well below the true value of 0.41. This verifies theoretical results on filter sensitivity [1]. The length of the residual sliding window is a design value and is a tradeoff between high P_d and low P_f. For $n_w=30$ and $P_f=0.1$ the limits for the sign test are 19 and 11 while the limits for r_1 are 0.27 and -0.33. For the GLR test the threshold was chosen by simulation to be $\lambda=15$.

As seen from Figs. 2-6 the proposed scheme performed quite well. The first stage detectors worked well indicating zero mean throughout but correlation soon after the onset of fault. The second stage GLR detector correctly identifies the fault and estimates the size and time of occurrence. However, more work should be done in the direction of a more methodical approach to the evaluation of the critical design parameters P_f, P_d, n_w and λ. Especially P_d is quite difficult to evaluate explicitly, since it is a function of the alternative hypothesis and depends on the true value of the failed parameter. We believe that simulation studies could solve this problem.

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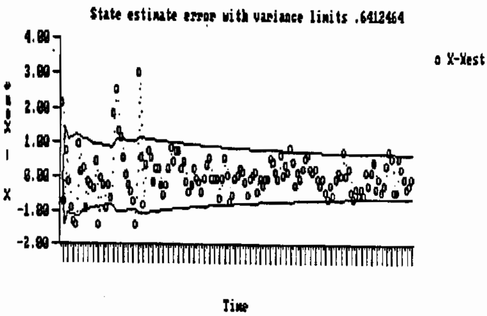

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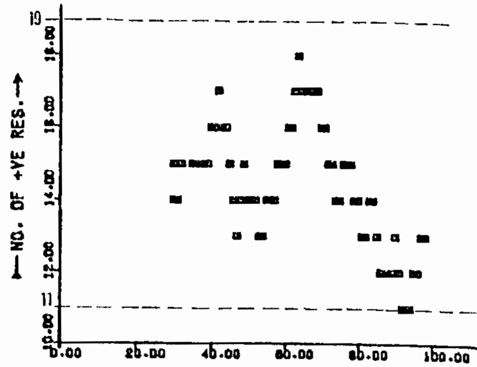

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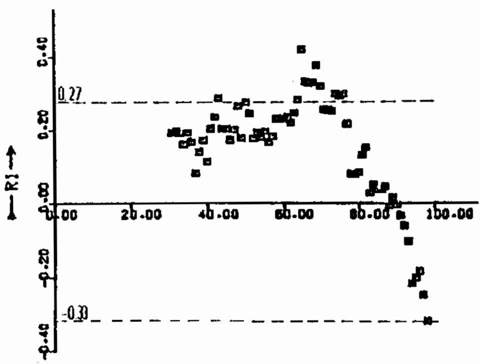

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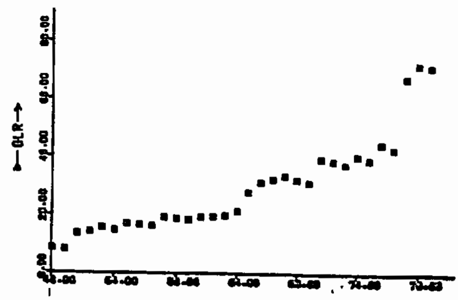

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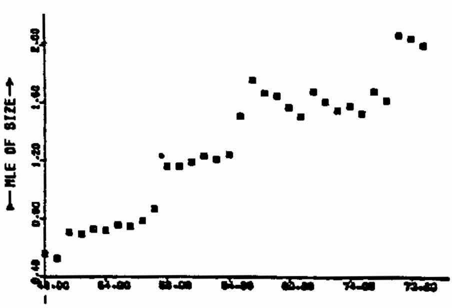

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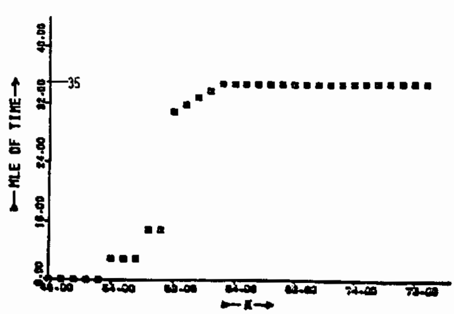

6

Figure 1. Filter estimate degradation due to additional sensor noise
Figure 2. Sign test with thresholds, n_W=30, P_f=0.1
Figure 3. First order serial correlation with thresholds, n_W=30, P_f=0.1
Figure 4. GLR value after first stage alarm (k=48)
Figure 5. GLR additional noise variance estimate
Figure 6. GLR failure time estimate