

Novelty of Frequency Domain Data in Smart Structures using μ -Analysis

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Abstract—This paper deals with the advantages of robust control in smart structures. First we present the implementations of H infinity control in the frequency domain. A dynamic model for smart structure under wind excitations is considered. Then robust control theory is used a model to synthesize controllers achieving stabilization with guaranteed performance for smart structures. We use μ -analysis to express the control problem as a mathematical optimization problem and then find the controller that solves the optimization problem in the frequency domain.

Index Terms—Frequency Domain, Robust Performance, Robust Stability, Smart Structures, Structural Control, M-Analysis.

I. INTRODUCTION

The trend of engineering design requires structures to become lighter, more flexible and stronger, so in recent years, the light structures have been widely used in various engineering applications [1]. The use of active control techniques for the suppression of vibrations of very light structures is a very important target in many applications, where the additional masses of stiffeners or dampers should be avoided. Active techniques are also more suitable in cases where the disturbance to be cancelled or the properties of the controlled system vary with time [2]. In practice, any structure that deforms under some loading can be regarded as flexible structure and is a distributed parameter system. This implies that vibration at one point is related to vibration at the rest of the points over the structure [3]. Therefore, in order to measure the complicates response of the structure and base on the control action, it is desirable to use appropriate sensors and actuators. Piezoelectric sensors and actuators are extensively employed in many practical applications such as smart structures due to their lightness and their capability of coupling strain and electric fields. In order to control structural vibrations, piezoelectric sensors and actuators can be easily bonded on the vibrating structure [4]-[6]. Robust control theory and m -analysis has the advantage over classical control techniques in that they are readily applicable to problems involving multivariate systems with cross-coupling between channels.

Simultaneously optimizing robust performance and robust stabilization is difficult. One method that comes close to achieving this is m -analysis, which allows the control designer to apply classical loop-shaping concepts to the multivariable frequency response to get good robust performance, and then optimizes the response near the system bandwidth to achieve good robust stabilization [7]. The Bode's integrals are used to approximate the derivatives of amplitude and phase of the plant model with respect to the frequency. Simulation examples illustrate the effectiveness and the simplicity of the proposed method to design the robust controllers [7]-[9].

This work is concerned with active vibration reduction of a smart beam, mounted rigidly along one edge to form a cantilever structure [10]-[12]. The beam, with piezoelectric sensor/actuator pairs bonded to its surfaces, is modeled using the super-convergent FE approach which includes extension, bending and rotation degrees of freedom [4], [5], [10]. For designing a controller, a structure consists of a cantilever beam with four surface bonded piezoelectric pairs is considered. The patches are used as actuators and sensors and they are attached symmetrically to either side of the beam, thus collocating the actuator and sensor. The parameters of the beam are shown in Table I. For the analysis of the cantilevered composite structure, a super-convergent finite element (FE) model is used [4], [6], [13], [14].

TABLE I: PARAMETERS OF THE SMART BEAM

Beam length, L	0.8m
Beam width, W	0.08m
Beam thickness, h	0.0093m
Beam density, ρ	1800kg/m ³
Young's modulus of the beam, E	1.5 X 10 ¹¹ N/m ²
Piezoelectric constant, d_{31}	254 X 10 ⁻¹² m/V
Electric constant, ξ_{33}	11.5 X 10 ⁻³ V m/N
Young's modulus of the piezoelectric element	1.5 X 10 ¹¹ N/m ²
Width of the piezoelectric element	$b_s = b_a = 0.07m$
Thickness of the piezoelectric element	$h_s = h_a = 0.0002m$

II. MODELLING

The dynamical description of the smart structure is given by,

$$M\ddot{q}(t) + D\dot{q}(t) + Kq(t) = f_m(t) + f_e(t) \quad (1)$$

Where M is the generalized mass matrix, D the viscous damping matrix, K the generalised stiffness matrix, f_m the external loading vector and f_e the generalised control force vector produced by electromechanical coupling effects. For

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a model simplified beam model of a composite beam with piezoelectric sensors and actuators the independent variable vector $q(t)$ is composed of transversal deflections w_i and rotations ψ_i , i.e for [10], [11], [15],

$$q(t) = \begin{bmatrix} w_1 \\ \psi_1 \\ \vdots \\ w_n \\ \psi_n \end{bmatrix},$$

where n is the number of finite elements used in the analysis. Vectors w and f_m are positive upwards.

To transform to state-space control representation, let (in the usual manner),

$$x(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix},$$

Furthermore to express $f_e(t)$ as $Bu(t)$ we write it as $f_e^* u$, where f_e^* is the piezoelectric force for a unit applied on the corresponding actuator, and u represents the voltages on the actuators. Lastly $d(t)=f_m(t)$ is the disturbance vector. Then,

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0_{2n \times 2n} & I_{2n \times 2n} \\ -M^{-1}K & -M^{-1}D \end{bmatrix} x(t) + \begin{bmatrix} 0_{2n \times n} \\ M^{-1}f_e^* \end{bmatrix} u(t) + \begin{bmatrix} 0_{2n \times 2n} \\ M^{-1} \end{bmatrix} d(t) \\ &= Ax(t) + Bu(t) + Gd(t) \\ &= Ax(t) + [BG] \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} \\ &= Ax(t) + \tilde{B}\tilde{u}(t) \end{aligned} \tag{2}$$

We can augment this with the output equation. For example, if we assume that displacements are only measured then,

$$y(t) = [x_1(t) x_3(t) \dots x_{n-1}(t)]^T = Cx(t) \tag{3}$$

With,

$$C = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & \vdots \\ & & \ddots & & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix},$$

In this formulation u is $n \times 1$ (at most, but can be smaller), while d is $2n \times 1$. The units used are m, rad, sec and N.

The control problem is to keep the beam in equilibrium (i.e. zero displacements and rotations) in the face of external disturbances, noise and model inaccuracies, using the available measurements (displacement) and controls [16], [12], [13].

III. FREQUENCY DOMAIN DATA

To relate the smart structure in frequency domain we use Fig. 1,

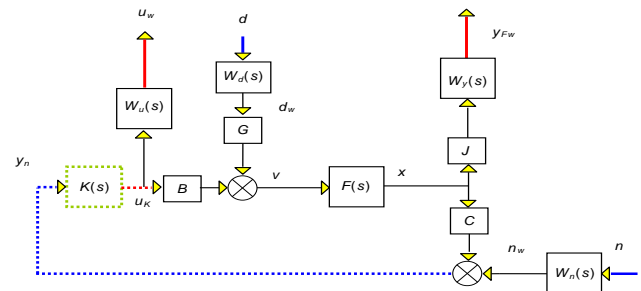


Fig. 1. Closed loop diagram in the frequency domain

In this diagram are included all inputs and outputs of interest, along with their respective weights.

d represent the external disturbances vector of our system (like wind or earthquakes), n is the noise of the system due the uncertainty of the model, B and G represent the matrices of (2), x is the state vector of the system, Ks is the controller, y represent the output vector and W are the necessary vectors of the weight, Wu for the controller, Wn for the noise, Wd for the disturbances and Wy for the outputs, and finally $F(s)$ is the necessary transfer function. We need to find this transfer functions [14], [16], [17]:

$$y_{Fw} = W_y J x = W_y J F v = W_y J F (G W_d d + B u_k) = W_y J F G W_d d + W_y J F B u_k$$

$$u_w = W_u u_k$$

$$y_n = C x + W_n n = C F v + W_n n = C F (G W_d d + B u_k) + W_n n = C F G W_d d + C F B u_k + W_n n$$

Combining all these gives,

$$\begin{bmatrix} u_w \\ y_{Fw} \\ y_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & W_u \\ W_y J F G W_d & 0 & W_y J F B \\ C F G W_d & W_n & C F B \end{bmatrix} \begin{bmatrix} d \\ n \\ u_k \end{bmatrix} \tag{4}$$

Note that the plant transfer function matrix, $F(s)$, is deduced from the suitably reformulated plant equations,

$$\dot{x}(t) = Ax(t) + Iv(t)$$

$$y(t) = Ix(t)$$

where $v(t)=Gd+Bu_k$. Hence,

$$F(s) = (sI - A)^{-1}$$

Using the H infinity control theory, the equivalent two-port diagram in the closed loop system is Fig. 2,

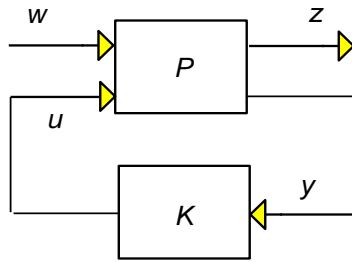


Fig. 2. Closed loop system, two port diagram

with,

$$z = \begin{bmatrix} u_w \\ y_{Fw} \end{bmatrix}, \quad w = \begin{bmatrix} d \\ n \end{bmatrix}, \quad y = y_n, \quad u = u_K$$

where z are the output variables to be controlled (the control vector and the state vector), and w the exogenous inputs (the disturbances and the noise vector)

Given that P has two inputs and two outputs it is, as usual, naturally partitioned as,

$$\begin{bmatrix} z(s) \\ y(s) \end{bmatrix} = \begin{bmatrix} P_{zw}(s) & P_{zu}(s) \\ P_{yw}(s) & P_{yu}(s) \end{bmatrix} \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} = P(s) \begin{bmatrix} w(s) \\ u(s) \end{bmatrix} \quad (5)$$

Also,

$$u(s) = K(s)y(s) \quad (6)$$

Using (4) the transfer function for P is,

$$P(s) = \begin{bmatrix} 0 & 0 & W_u \\ W_y JFGW_d & 0 & W_y JFB \\ \hline CFGW_d & W_n & CFB \end{bmatrix} \quad (7)$$

while the closed loop transfer function $M_{zw}(s)$ is,

$$M_{zw}(s) = P_{zw}(s) + P_{zu}(s)K(s)(I - P_{yu}(s)K(s))^{-1}P_{yw}(s) \quad (8)$$

Equation (8) is the well-known lower LFT for M_{zw} .

To express P in state space form, the natural partitioning [18]-[20],

$$P(s) = \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right] = \begin{bmatrix} P_{zw}(s) & P_{zu}(s) \\ P_{yw}(s) & P_{yu}(s) \end{bmatrix} \quad (9)$$

is used (where the packed form has been used, while the corresponding form for K is,

$$K(s) = \left[\begin{array}{c|c} A_K & B_K \\ \hline C_K & D_K \end{array} \right]$$

Equation (10) defines the equations,

$$\dot{x}(t) = Ax(t) + \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} w(t) \\ u(t) \end{bmatrix}$$

$$\begin{bmatrix} z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x(t) + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} w(t) \\ u(t) \end{bmatrix}$$

and,

$$\dot{x}_K(t) = A_K x_K(t) + B_K y(t)$$

$$u(t) = C_K x_K(t) + D_K y(t)$$

To find the matrices involved, we break the feedback loop and use the relevant equations.

Therefore, the equations relating the inputs, outputs, states and input/output to the controller are [14], [15]:

$$\begin{aligned} \dot{x}_F &= Ax_F + (Gd_w + Bu), & y_F &= x_F \\ \dot{x}_u &= A_u x_u + B_u u, & u_w &= C_u x_u + D_u u \\ \dot{x}_{yF} &= A_{yF} x_{yF} + B_{yF} J y_F, & y_{Fw} &= C_{yF} x_{yF} + D_{yF} y_F \\ \dot{x}_n &= A_n x_n + B_n n, & n_w &= C_n x_n + D_n n \\ \dot{x}_d &= A_d x_d + Gd, & d_w &= C_d x_d + D_d d \\ y_n &= C y_F + n_w \end{aligned} \quad (10)$$

$$x = \begin{bmatrix} x_F \\ x_u \\ y_{Fw} \\ x_n \\ x_d \end{bmatrix}, \quad y = y_n, \quad w = \begin{bmatrix} d \\ n \end{bmatrix}, \quad z = \begin{bmatrix} u_w \\ y_{Fw} \end{bmatrix}, \quad u = u_K$$

By taken d_w , n_w και y_F from (11) we have,

$$\dot{x} = \begin{bmatrix} A_G & 0 & 0 & 0 & GC_d \\ 0 & A_u & 0 & 0 & 0 \\ BC_F & 0 & A_{yF} & 0 & 0 \\ 0 & 0 & 0 & A_n & 0 \\ 0 & 0 & 0 & 0 & A_d \end{bmatrix} x + \begin{bmatrix} GD_d & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & B_n \\ B_d & 0 \end{bmatrix} w + \begin{bmatrix} B \\ B_u \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$

$$z = \begin{bmatrix} 0 & C_u & 0 & 0 & 0 \\ D_{yF} C_F & 0 & C_{yF} & 0 & 0 \end{bmatrix} x + 0w + \begin{bmatrix} D_u \\ 0 \end{bmatrix} u$$

$$y = [C_F \ 0 \ 0 \ C_n \ 0]x + [0 \ D_n]w + 0u$$

Therefore, the matrices are:

$$A_1 = \begin{bmatrix} A_F & 0 & 0 & 0 & GC_d \\ 0 & A_u & 0 & 0 & 0 \\ BC_F & 0 & A_{yF} & 0 & 0 \\ 0 & 0 & 0 & A_n & 0 \\ 0 & 0 & 0 & 0 & A_d \end{bmatrix}, \quad B_1 = \begin{bmatrix} GD_d & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & B_n \\ B_d & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} B \\ B_u \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & C_u & 0 & 0 & 0 \\ D_{yF} C_F & 0 & C_{yF} & 0 & 0 \end{bmatrix}, D_{11} = 0, D_{12} = \begin{bmatrix} D_u \\ 0 \end{bmatrix}$$

$$C_2 = [C_F \ 0 \ 0 \ C_n \ 0], D_{21} = [0 \ D_n], D_{22} = 0$$

IV. EXTERNAL DISTURBANCES

We use the wind force in our smart structures Fig 3. The wind load is a real life wind speed measurement in relevance with time that took place of Heraklion Crete. We transform the wind speed in wind pressure with, loading corresponds to the wind excitation. The function $d(t) = f_m(t)$ has been obtained from the wind velocity record, through the relation

$$f_m(t) = \frac{1}{2} \rho C_u V^2(t) \tag{17}$$

where V =velocity, ρ =density and $C_u=1.5$ (orthogonal cross-section)

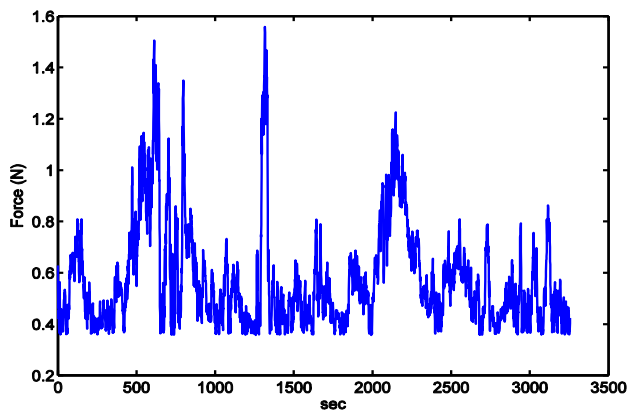


Fig. 3. Wind load

Moreover, in all simulations, random noise has been introduced to measurements at system output locations within a probability interval of $\pm 1\%$. Due to small displacements of system nodal points, noise amplitude is taken to be small, of the order of 5×10^{-5} of the initial prices. On the other hand, the signal is introduced at each node of the beam by a different percentage, that percentage being lower at the first node due to the fact that the beam end point is clamped [12], [15], [18].

A. Results without Weights

In the simplest approach no weights are placed on any of the input/output quantities. This means that the H infinity (H_∞) controller ensures [20]-[22],

$$\left\| \begin{bmatrix} e \\ u \end{bmatrix} \right\|_2 \leq 1$$

As long as,

$$\left\| \begin{bmatrix} d \\ n \end{bmatrix} \right\|_2 \leq 1$$

Figs. 4-5 show the results of this run.

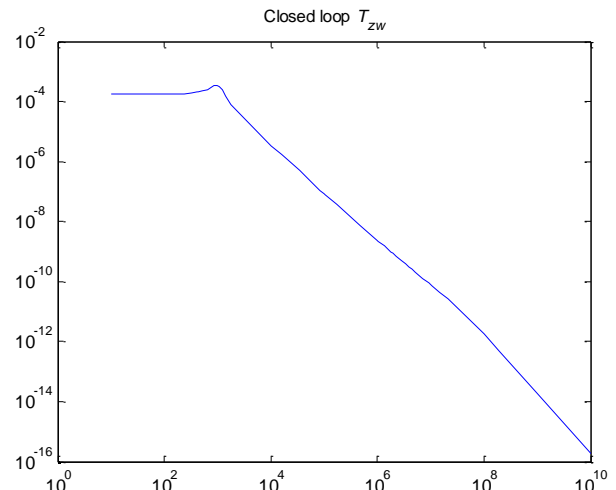


Fig. 4. Closed loop T_{zw} for all frequencies

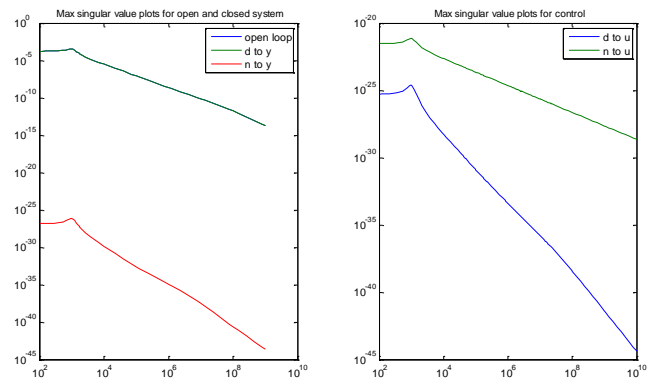


Fig. 5. Max singular values for closed and open loop

Fig. 4 shows that the price of the singular value of the unweighted system is very small for all frequencies (much lower than one). Fig. 5 shows a satisfactory effect of the disturbance on the size of the control scheme (the design could be improved, if it were possible to reduce noise effect for frequencies of 1000 Hz). There are no difference is observed between the frequency plots of open and closed loop for the unweighted system.

B. Results with weights

Next we try constant weights, in particular let,

$$W_n = 10^{-8}, W_u = 1/500, W_e = 10^3$$

Figs. 6-7 promises a marked improvement in performance Fig.6 shows that the value of T_{zw} is low then one for all frequencies.

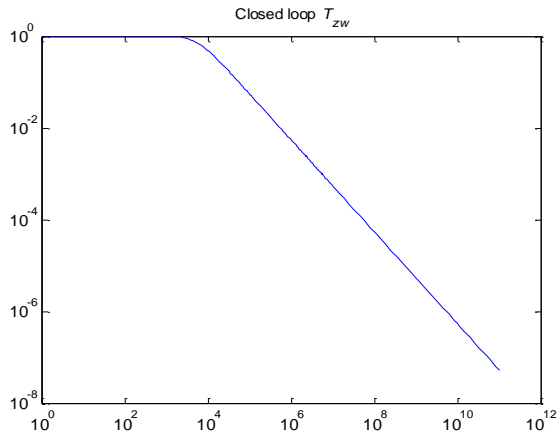


Fig. 6. Closed loop T_{zw} for all frequencies

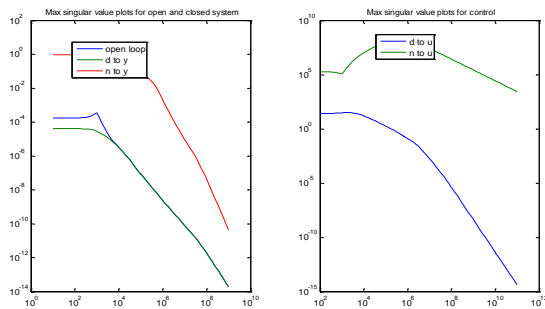


Fig. 7. Max singular values for closed and open loop

Fig. 7 shows a satisfactory effect of the disturbance on the size of the control scheme (the design could be improved, if it were possible to reduce noise effect for frequencies of 1000 Hz). There are no difference is observed between the frequency plots of open and closed loop system.

As shown in Fig. 7a, there is a significant improvement in the effect of disturbance on error up to the frequency of 1000 Hz. In Fig. 13a, there seems to be little effect of noise on error for frequencies beyond 1000 Hz. In Fig. 7b, shows a satisfactory effect of the disturbance on the size of the control scheme (the design could be improved, if it were possible to reduce noise effect for frequencies of 1000 Hz).

V. ROBUSTNESS ISSUES

The superiority of *H infinity* control lies in its ability to take explicitly into account the worst effect of unknown disturbances and noise in the system. Furthermore, at least in theory, it is possible to synthesize a controller that is robust to a prescribed amount of modeling errors. Unfortunately, this last possibility is not implementable in

some cases, as it will be subsequently illustrated [19]-[21].

In what follows, the robustness to modeling errors of the designed *H infinity* controller will be analyzed. Furthermore, an attempt to synthesize a μ -controller will be presented, and comparisons between the two will be made [21].

Numerical models used in all simulations, are implemented in three ways [18]:

1. Through (11),

$$K = K_0 (I + k_p \delta_K)$$

$$M = M_0 (I + m_p \delta_M)$$

(11)

$$D = D_0 + 0.0005 [K_0 k_p I_{2n \times 2n} \delta_K + M_0 m_p I_{2n \times 2n} \delta_M]$$

and subsequent evaluation of matrix N for specific values of k_p , m_p from zero to one.

K_0 and M_0 represent the nominal vector of the Stiffness and the Mass matrices, I is the identity matrix, δ_M and δ_K represent the uncertainty. The norm of uncertainty must be less than one. Alternatively, since in general,

$$D = \alpha K + \beta M$$

D could be expressed similarly to K , M , as,

$$D = D_0 (I + d_p I_{2n \times 2n} \delta_D)$$

In this way we introduce uncertainty in the form of percentage variation in the relevant matrices. This expression for uncertainty is suitable in our case, uncertainty is most likely to arise from terms outside the main matrices (since length can be adequately measured).

Here it will be assumed,

$$\| \Delta \|_Y \stackrel{def}{=} \left\| \begin{bmatrix} I_{n \times n} \delta_K & 0_{n \times n} \\ 0_{n \times n} & I_{n \times n} \delta_M \end{bmatrix} \right\|_Y < 1$$

hence m_p , k_p are used to scale the percentage value and the zero subscript denotes nominal values.

2. By use of Matlab's "uncertain element object". As explained, this form is needed in the D-K robust synthesis algorithm.

3. By Simulink implementation of Fig. 8,

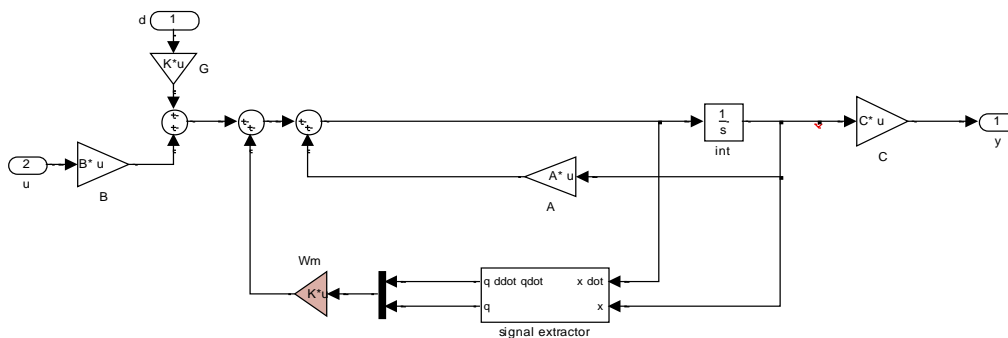


Fig. 8. Simulink diagram of uncertain plant

Robust analysis is carried out through the relations:

$$\sup_{\omega \in \Omega} \mu_{\Delta} (N_{11}(j\omega)) < 1 \quad (12)$$

(for robust stability), and,

$$\sup_{\omega \in \Omega} \mu_{\Delta_u} (N(j\omega)) < 1 \quad (13)$$

for robust performance.

In all the simulations that follow the disturbance is the mechanical load 10N at the free end of the structures.

VI. RESULTS FOR μ -ANALYSIS

First of all we take $m_p=0, k_p=0.9$. This corresponds to a $\pm 90\%$ variation from the nominal value of the stiffness matrix K.

In Fig. 9 are shown the displacement responses for this controller for the first mechanical input. In Fig. 10 are shown the bounds on the μ values. As seen the system remains stable and exhibits robust performance, since the upper bounds of both values remain below 1 for all frequencies of interest. This result is validated in Fig. 11, where the displacement of the free end and the voltage applied are shown at the extreme uncertainty. Comparison with the open loop response for the same plant shows the good performance of the nominal controller.

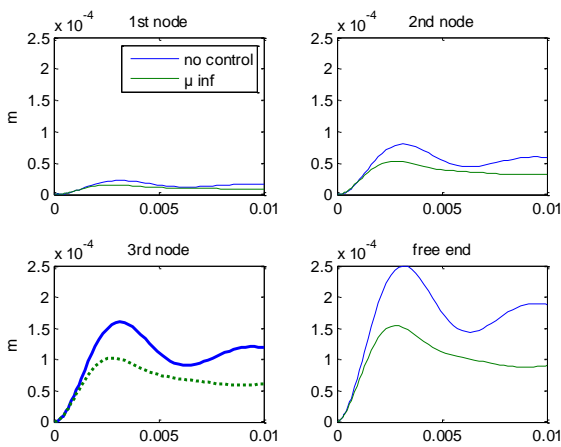


Fig. 9. Displacement response, 10N at free end, μ -controller for $m_p=0, k_p=0.9$

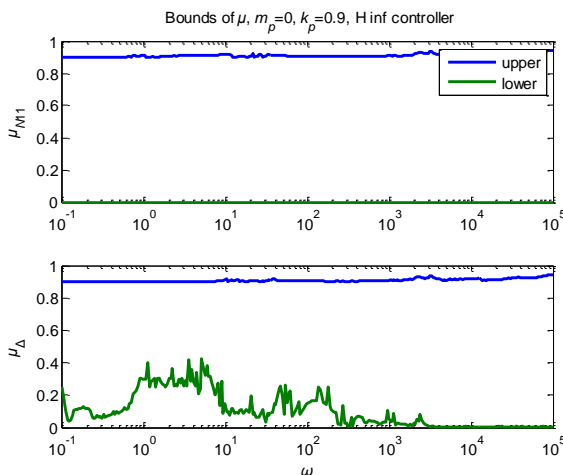


Fig. 10. μ -bounds of the H_{∞} controller for $m_p=0, k_p=0.9$

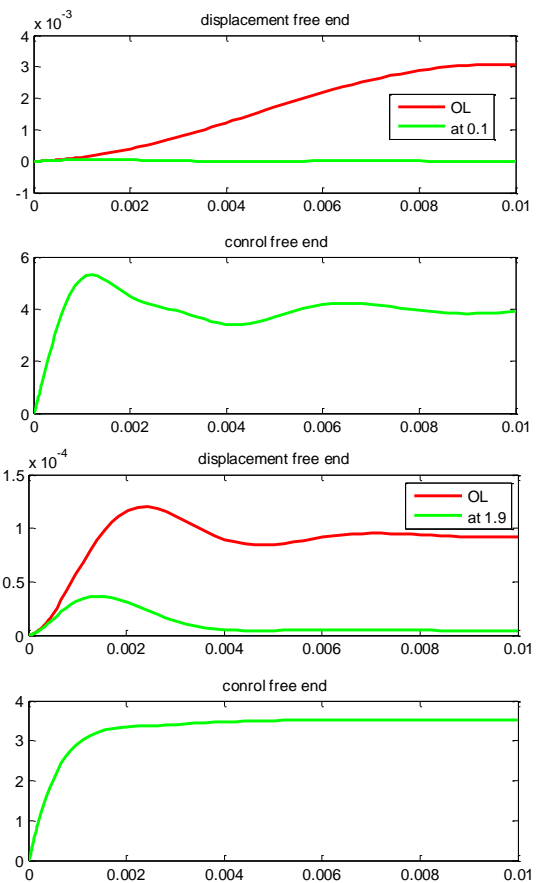


Fig. 11. Displacement and control at free end for the H_{∞} controller with $m_p=0, k_p=0.9$ (extreme values)

VII. RESULTS FOR ROBUST SYNTHESIS: M-CONTROLLER

A μ -controller can be synthesized via the procedure of D-K iteration. As explained, this is an approximate procedure, providing bounds on the μ -value. To facilitate comparison with the H_{∞} controller, similar bounds for the uncertainty will be used [14], [15].

I. $m_p=0, k_p=0.9$. This corresponds to a $\pm 90\%$ variation from the nominal value of the stiffness matrix K. where $A0_u, B0_u$ and $G0_u$ are uncertain matrix objects.

This command produces a robust controller of order 256. This is an enormous value, which is a result of the way this algorithm works. However, even though this fact is mentioned in the literature, it is not given the appropriate attention, and is definitely a shortcoming. To our knowledge, there is no easy way to lower the order, unless a tedious manual approach is used.

In Fig. 12 μ -values of the calculated controller are shown. As seen the controller is robust in most frequencies.

In Fig. 13 performance of the μ and H_{∞} controllers is compared at the free end (this is indicative of overall performance). As seen the H_{∞} controller performs better at the expense of increased control effort. Fig. 14 (left window) verifies this result, where it is seen that the H_{∞} controller fares better at the extreme value. This could be due to numerical difficulties in the calculation of the μ -controller arising from the bad condition number of the plant. It could also be due to the high order of the μ -controller. In any case, further investigation is needed.

VIII. CONCLUSIONS

In this paper, a robust control design problem has been formulated within a linear fractional transformation framework using the *Hinfinity* and μ -analysis technique. A suboptimal controller has been used for numerical modeling. The open loop and the closed-loop controlled system has been simulated using a periodic impulsive command input, periodic isolated influences. The mathematical model derived using robust control is compared with models obtained by more conventional and well known methods. Using this model, a *Hinfinity* controller is designed for vibration suppression purposes. An optimal controller is the trained using nonconvex and nonsmooth optimization to mimic the previous controller, μ -analysis technique has the advantage over classical control techniques in that they are readily applicable to problems involving multivariate systems with cross-coupling between channels. Simultaneously optimizing robust performance and robust stabilization is difficult. One method that comes close to achieving this is μ -analysis, which allows the control designer to apply classical loop-shaping concepts to the multivariable frequency response to get good robust performance, and then optimizes the response to achieve good robust stabilization.

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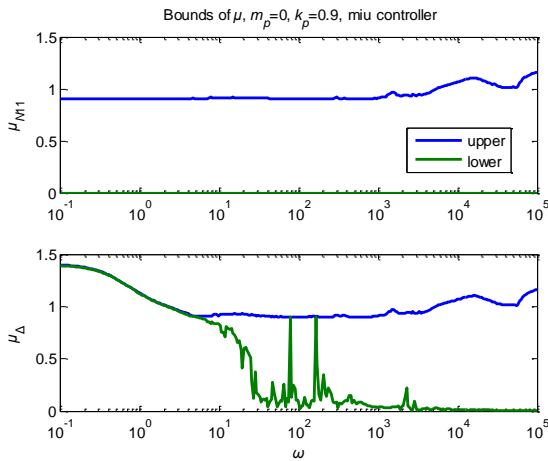


Fig. 12. μ -bounds of the μ -controller for $m_p=0, k_p=0.9$

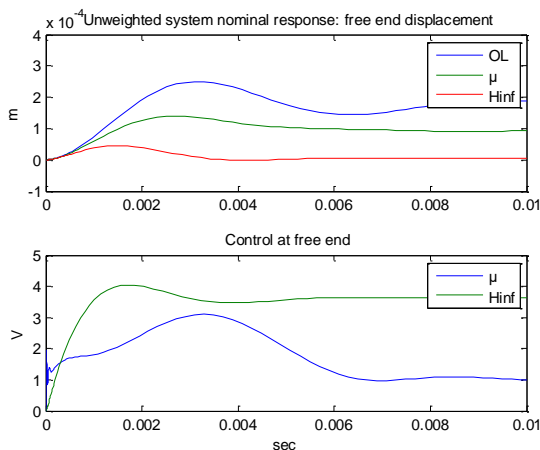


Fig. 13. Comparison of free end data of nominal system for μ -controller ($m_p=0, k_p=0.9$) and H^∞

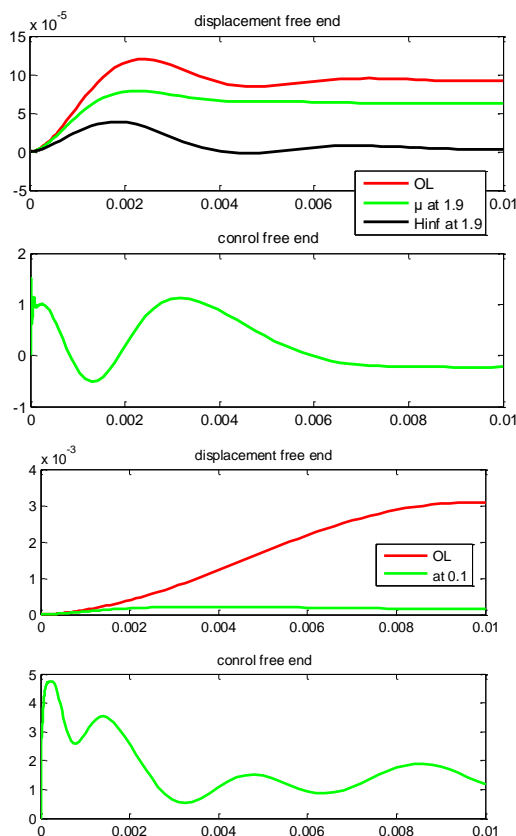


Fig. 14. Displacement and control at free end for the μ -controller with $m_p=0, k_p=0.9$ (extreme values)

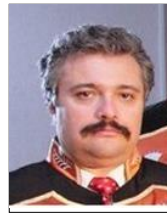
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